# Commercial Policies in the Presence of Input-Output 

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#### Abstract

How do input-output linkages modify countries' incentives to optimally conduct commercial policies? We address this question in a version of the Melitz (2003) model where the production-side of the economy is enriched by input-output linkages. In addition to labor, production of differentiated varieties requires a bundle of intermediate inputs, which is a composite good governed by the same CES aggregator as the final good. Optimal cooperative policies of potentially asymmetric countries correct for an input distortion generated by the fact that firms' monopolistic markups translate into a distorted price of the composite intermediate input. In the analysis of optimal non-cooperative trade policy, the input distortion stemming from domestic markups counteracts the standard terms-of-trade externality, reducing the optimal import tariff and potentially turning it into an import subsidy, if the input distortion is large.


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[^0]
## 1 Introduction

One of the salient features of modern globalization is the high and increasing share of trade in intermediate inputs. The content of imported inputs in world exports was estimated at over $30 \%$ for 1995 (Hummels et al., 2001) and has risen since. The value-added to grossvalue ratio of exports, an inverse measure of vertical specialization, has fallen since 1970, with the strongest decline taking place since the 1990s (Johnson and Noguera, 2012). Antràs et al. (2012) document a rise in the "upstreamness" of industries, another measure of vertical integration, in particular in the 2000s. A large share of the increase in world trade over the past two decades represents trade in intermediate inputs. In this paper we demonstrate that in the "work-horse" model of trade with monopolistic competition and firm heterogeneity (Melitz, 2003) the presence of input-output linkages per se generates a distortion and, thus, a case for policy intervention that has so far not been considered in the literature.

Recent literature has emphasized that trade of, or in the presence of, intermediate inputs is different from trade in final goods. Yi (2003) argues that input-output linkages magnify the effects of trade policy. ${ }^{1}$ Putting numbers on the welfare formula developed by Arkolakis et al. (2012), Costinot and Rodríguez-Clare (2014) show that, conditional on observed trade shares and the estimated trade elasticity, implied welfare gains from trade are larger by an order of magnitude if one accounts for input-output linkages compared to the case of trade in final goods alone. Theoretical work by Jones (2000) and Grossman \& Rossi-Hansberg (2008), among others, has identified productivity effects of intermediate input trade that serve to explain these magnified numbers. Antràs (2015) emphasizes trade in intermediate inputs is far more likely to suffer from contractual imperfections, due to relationship specificity and limited third party verifiability of product characteristics. Antràs \& Staiger (2012), Díez (2014) and Ornelas \& Turner (2012) identify novel arguments for specific policy interventions that derive from trade distortions generated by such contractual imperfections.

In this paper we explore the policy implications of input-output linkages in a world with perfect contracts and monopolistic competition among heterogeneous firms, as modeled in

[^1]Melitz (2003). It is well known that without input-output linkages the decentralized equilibrium in this world features efficient firm entry and production (Dhingra and Morrow, 2014). This holds true for a closed as well as for a perfectly integrated world economy. It also holds true in the case of costly trade, provided one looks at joint welfare of all countries considered. ${ }^{2}$ From a unilateral perspective, an import tariff is optimal. ${ }^{3}$ In this paper, we analyze optimal commercial policies in a version of the Melitz (2003) model where the production-side of the economy is enriched by input-output linkages. Such a model extension has already been presented in Costinot and Rodríguez-Clare (2014) and Caliendo et al. (2015), but these authors do not consider commercial policies. We are prompted to consider optimal commercial policies by a very simple idea. If intermediate inputs are produced under monopolistic competition their price is above marginal cost, and, unless primary inputs are similarly priced above marginal cost, this creates an input distortion. For instance, assuming well-functioning markets for primary inputs, firms will end up using too much of primary inputs, and too little of intermediate inputs.

We address the policy conclusions that derive from this input distortion in stylized model that allows us to squarely focus on this distortion. We assume an endogenous mass of monopolistically competitive producers of differentiated varieties use a Cobb-Douglas technology that combines labor and a bundle of intermediate inputs. The bundle of intermediates is a composite good governed by the same CES aggregator as the final good, which is assembled by perfectly competitive firms. Firms differ in productivity as in Melitz (2003), where a zero expected profit condition determines the mass of entering firms and a zero profit condition determines the mass of successive entry into domestic and export markets. To put into sharp focus the interaction between input-output linkages and intra-industry reallocation, we assume a single-sector economy. ${ }^{4}$ Compared to the standard Melitz (2003) model where intermediate input producers set profit-maximizing prices, our model gives them an

[^2]additional margin of adjustment, viz. the choice of using labor as a direct input or draw on labor indirectly through the use of intermediate inputs. Firms make this choice in pursuit of cost minimization, but due to the aforementioned input distortion firm behavior leads to production inefficiency in the sense of too little intermediate input use. ${ }^{5}$ From an industry perspective, firm choices affect the allocation of the composite good to intermediate and final use.

The input distortion provides a rationale for welfare-enhancing policy intervention. We assume that governments conduct commercial policies modeled as wedges between consumer prices and ex factory prices for domestically produced and imported varieties (inclusive of real trade cost), thereby extending Caliendo et al. (2015) to the cases of import subsidies as well as domestic consumption tax-cum-subsidies. ${ }^{6}$ Net government revenue is fully redistributed to workers in a lump-sum fashion. Subsidies are financed by lump-sum taxes on labor. The optimal cooperative policy that addresses the input distortion is a subsidy on domestically and imported intermediate goods that exactly offsets the markup. ${ }^{7}$ The optimal subsidy rate is independent of the degree of firm-level heterogeneity in productivities and of the labor cost share in production. The result carries over to case of asymmetric countries.

We also analyze optimal cooperative trade policy. Clearly, with only trade policy at their disposal governments cannot implement first-best outcomes. The optimal cooperative policy is an import subsidy as in the case of uniform treatment of domestic and imported intermediates. The optimal subsidy rate (in absolute terms) is the larger, the smaller the labor cost share. Intuitively, with a low labor cost share the input distortion is more severe, calling for a stronger policy invention. Compared to the optimal uniform subsidy, the optimal subsidy

[^3]rate for imports is smaller if the labor cost share is sufficiently low, and larger if the labor cost share is sufficiently small. In general, policy intervention is less efficient in addressing the input distortion when the freeness of trade is low. Intuitively, with a low freeness of trade, the fraction of imported in available varieties is small, diminishing the impact of import subsidies on the aggregate price of the bundle of intermediate goods. The optimal import subsidy rate is closer to zero, if freeness of trade is low.

Finally, we look at noncooperative trade policy. From a domestic perspective, while domestic intermediate goods come at prices above their true domestic opportunity costs (as in the case of cooperative policies), the relevant domestic opportunity costs for imported intermediates are given by their border prices, notwithstanding the fact these prices are above the foreign opportunity costs. This observation has three implications. First, the input distortion remains, but its relevance depends on labor cost share and the weight of domestic goods in the composite intermediate input. Due to the asymmetry across domestic and imported varieties, a uniform treatment cannot be optimal. Second, an import subsidy will lower the aggregate price of the intermediate input bundle, but is not the first-best policy to address the input distortion. ${ }^{8}$ Third, governments will find it optimal to exploit the terms-of-trade externality by means of an import tariff (or domestic subsidy), as in the standard model, but clearly, the input distortion and the terms-of-trade externality call for opposing policy interventions. We find that the optimal import tariff, if any, will never be larger in the presence of input-output linkages than without. Intuitively, the presence of the input distortion calls for a more careful use of a tariff. If the labor cost share is sufficiently large, the optimal tariff is increasing in the freeness of trade, a result that generalizes Felbermayr et al. (2013). The input distortion dominates, rendering an import subsidy optimal, if the labor cost share is sufficiently low and the freeness of trade sufficiently large. Reducing the freeness of trade for a given labor cost share leads to import subsidies closer to zero for the same reason as in the cooperative trade policy case. A further reduction in the freeness of trade potentially results an optimal import tariff, as the terms-of-trade consideration starts to dominate again.

[^4]In addition to the literature mentioned above, this paper more directly relates to the following papers. Our optimal trade policy results generalize the findings in Felbermayr et al. (2013) to the presence of input-output linkages. ${ }^{9}$ Costinot et al. (2016) extend a framework similar to ours in several directions, but do not consider input-output linkages. Blanchard et al. (2016) use a terms-of-trade model of trade policy with political economy motives. They allow for tariffs on final goods, but rule out trade taxes/subsidies for intermediate goods. They find that final good tariffs decrease in the domestic content of foreign-produced final goods. We work with a different theoretical framework and consider another polar case where governments cannot distinguish between intermediate and final use. Caliendo et al. (2015) work with a multi-sector version of the present framework, but only consider the welfare consequences of import tariffs. We discuss the underlying distortion and characterize optimal cooperative and non-cooperative policies.

From a more broader perspective, we contribute to the literature on efficiency of market outcomes when one deviates from the standard assumptions of a monopolistic competition model. Dhingra and Morrow (2014) analyze deviations on the demand-side of the economy. They postulate a demand function that generates variable markups and discuss the implications for efficiency. Jung (2015) analyzes allocational efficiency in a Melitz-type model with CES-Benassy preferences, which allows for disentangling love of variety and market power. He shows that when the strength of love of variety is lower than implicitly assumed under standard CES preferences, product variety is too large and productivity is too low. The firstbest policy is then a tax on (operating) production fixed costs. In the present paper, we stick to CES demand, but analyze the implications of modifications on the production-side of the economy for efficiency and optimal policies.

The remainder of the paper is structured as follows. Section 2 introduces commercial policies into a version of the Melitz (2003) model that allows for intermediate inputs and input-output-linkages and derives a generalized welfare formula. Section 3 derives optimal

[^5]cooperative policies of potentially asymmetric countries. Section 4 characterizes welfaremaximizing non-cooperative trade policy. The final section concludes.

## 2 Model

In this section we generalize Caliendo et al. (2015) by introducing a tax-cum-subsidy on the "consumption" of domestically produced and imported intermediate inputs. ${ }^{10}$ The model features monopolistic competition among differently productive firms, as in Melitz (2003), the key novel feature being that production requires intermediate inputs in addition to labor. For the sake of tractability and easier notation, we assume that the bundle of intermediate inputs used in production is a composite good governed by the same CES aggregator as the final good. This captures, albeit in a stylized way, the fact a large class of goods are used use in consumption as well as intermediate inputs in production. We assume, plausibly, that when imposing taxes or subsidies governments cannot distinguish between "final use" and "input use" of a good. ${ }^{11}$

There is a world with $M$ countries, indexed by $i$ and $j$. Within each country, the final good is assembled from intermediate goods originating in all countries, and production of any intermediate good uses labor as well as the aforementioned composite of intermediate inputs. We use the term intermediate goods when referring to assembly of final goods, and the term intermediate input when referring to production of intermediate goods. Labor markets in all countries as well as final goods markets are perfect whereas markets for intermediate goods are characterized by monopolistic competition. The "number" of intermediate goods produced in each country is determined by entry of firms, subject to a fixed entry cost and the usual productivity draw from a distribution function $G(\varphi)$ - with corresponding density $g(\varphi)$ - which is assumed Pareto and the same for all countries. We use $N_{j}$ to measure the mass of entrants, i.e. potential producers, in country $j$. In addition to a fixed cost $f_{j i}>0$ of taking

[^6]up shipments of a good produced in country $j$ to country $i$, there are "iceberg costs" of trade, $\tau_{j i} \geq 1$.

Our paper focuses on optimal government intervention in order to correct distortions that are present in this model. We shall look at efficient policies, formed in a cooperative way by all countries, as well as non-cooperative policies. The policy instruments considered will be taxes/subsidies as well as tariffs.

### 2.1 Households

We assume that a representative household in any one country $i$ consumes a composite good that is assembled from tradable intermediates originating in all of the $M$ countries. Utility is linear in consumption of this good, denoted by $C_{i}$ :

$$
\begin{equation*}
U_{i}\left(C_{i}\right)=C_{i} . \tag{1}
\end{equation*}
$$

Assembly is governed by a CES aggregator with elasticity of substitution $\sigma>1$. Given that in each country the market for this good is perfectly competitive, the good is sold at a price $\tilde{P}_{i}$, which is equal to the minimum unit cost, given prices of the intermediates, inclusive of iceberg-type trade costs and taxes or subsidies.

Households derive income from two sources. They in-elastically supply one unit of labor, earning a wage equal to $w_{i}$. In addition, they receive a lump-sum redistribution of any revenue generated by government policies, or face a lump-sum tax such as might be required to finance the fiscal cost of such policies. Denoting the lump-sum transfer/tax in country $i$ by $T_{i}$, aggregate income by households in country $i$ is

$$
\begin{equation*}
I_{i}=w_{i} L_{i}+T_{i}, \tag{2}
\end{equation*}
$$

where $L_{i}$ denotes the mass of households/consumers in country $i$. Given perfect labor markets, firms as well as households assume $w_{i}$ to be given. Moreover, we assume households to spend all income on consumption of the final good.

### 2.2 Assembly of the aggregate good

We use $Q_{i}$ to denote the sum of $C_{i}$ plus plus demand for this same type of aggregate good to be used as an intermediate input in the production of all firms located in country $i$. As indicated above, we assume that assembly of this good is governed by a CES production function with elasticity of substitution $\sigma>1$. Thus, we have

$$
\begin{equation*}
Q_{i}=\left[\sum_{j=1}^{M} N_{j} \int_{\varphi_{j i}^{*}}^{\infty} \tilde{q}_{j i}(\varphi)^{\frac{\sigma-1}{\sigma}} g(\varphi) \mathrm{d} \varphi\right]^{\frac{\sigma}{\sigma-1}} . \tag{3}
\end{equation*}
$$

In this expression, $\tilde{q}_{j i}(\varphi)$ denotes the quantity of an intermediate good originating in country $j$ and produced by a firm with productivity $\varphi$, available for use in assembly of the aggregate good in country $i$. Note that we assume a uniform elasticity of substitution for all countries. Assuming costless product differentiation and modeling firm heterogeneity with a continuum of firms, we may use $\varphi$ to index varieties of intermediates, or firms, whereby $\varphi_{j i}^{*}$ denotes the threshold that a firm operating in country $j$ needs to surpass to profitably sell its product in country $i$, given iceberg trade costs and fixed market access costs.

Cost-minimizing assembly requires

$$
\begin{equation*}
\min _{\left\{\tilde{q}_{j i}(\varphi)\right\} \geq 0} \sum_{j=1}^{M} N_{j} \int_{\varphi_{j i}^{*}}^{\infty} \tilde{p}_{j i}(\varphi) \tilde{q}_{j i}(\varphi) g(\varphi) \text { d } \varphi \text { s.t. equation (3). } \tag{4}
\end{equation*}
$$

In this expression, $\tilde{p}_{j i}(\varphi)$ denotes the price of a good originating in country $j$, produced by a firm with productivity $\varphi$, and sold country $i$, inclusive of iceberg trade costs and wedges introduced by government policies of country $i$. Given perfect competition on all national markets for the aggregate good, the value function corresponding to this minimization problem, per unit of $Q_{i}$, is equal to the price of this good which we denote by $\tilde{P}_{i}$ :

$$
\begin{equation*}
\tilde{P}_{i}=\left(\sum_{j=1}^{M} N_{j} \int_{\varphi_{j i}^{*}}^{\infty} \tilde{p}_{j i}(\varphi)^{1-\sigma} g(\varphi) \mathrm{d} \varphi\right)^{\frac{1}{1-\sigma}} . \tag{5}
\end{equation*}
$$

Using $Y_{i}:=\tilde{P}_{i} Q_{i}$ to denote the value of aggregate demand in country $i$, conditional demand
of country $i$ for a variety $\varphi$ from country $j$ follows as

$$
\begin{equation*}
\tilde{q}_{j i}(\varphi)=\left(\frac{\tilde{p}_{j i}(\varphi)}{\tilde{P}_{i}}\right)^{-\sigma} \frac{Y_{i}}{\tilde{P}_{i}} . \tag{6}
\end{equation*}
$$

### 2.3 Production

Producers of goods used for assembly use two types of inputs: labor, denoted by $l$, and a bundle of intermediate inputs, denoted by $m .^{12}$ For simplicity, we assume technology to be symmetric across all firms and countries. As indicated above, the bundle $m$ is composed of intermediates according the exact same CES aggregate that governs final assembly and is given in (3). We thus assume that any producer can source intermediate inputs from abroad without paying additional fixed costs, over and above the variable iceberg cost which also govern imports for the purpose of final goods assembly. ${ }^{13}$ Labor and the intermediate input bundle are combined using the following constant returns to scale Cobb-Douglas production function

$$
\begin{equation*}
\tilde{q}_{j}(\varphi)=\varphi l_{j}(\varphi)^{\gamma} m_{j}(\varphi)^{1-\gamma} . \tag{7}
\end{equation*}
$$

In this expression, $l_{j}(\varphi)$ denotes labor input and $m_{j}(\varphi)$ denotes the quantity of the aggregate bundle of intermediate inputs used by firm $\varphi$ located in country $j$. Note that the production function (7) nests the standard Melitz (2003) case without input-output linkages for $\gamma=1$.

Cost minimization by firm $\varphi$ requires

$$
\begin{equation*}
\min _{\left(l_{i}(\varphi), m_{i}(\varphi)\right) \geq 0}\left\{w_{j} l_{j}(\varphi)+\tilde{P}_{j} m_{j}(\varphi)\right\} \quad \text { s.t. equation (7). } \tag{8}
\end{equation*}
$$

Conditional input demands emerge as

$$
\begin{equation*}
l_{j}(\varphi)=\gamma \frac{x_{j}}{w_{j}} \frac{\tilde{q}_{j}(\varphi)}{\varphi} \quad \text { and } \quad m_{j}(\varphi)=(1-\gamma) \frac{x_{j}}{\tilde{P}_{j}} \frac{\tilde{q}_{j}(\varphi)}{\varphi}, \tag{9}
\end{equation*}
$$

[^7]where $x_{j}$ denotes the value function corresponding to (8), per unit of $\frac{\tilde{q}_{j}(\varphi)}{\varphi}$. It is straightforward to show that
\[

$$
\begin{equation*}
x_{j}=A w_{j}^{\gamma} \tilde{P}_{j}^{1-\gamma}, \quad \text { where } \quad A:=\gamma^{-\gamma}(1-\gamma)^{\gamma-1} . \tag{10}
\end{equation*}
$$

\]

Marginal cost of firm $\varphi$ in country $j$ is equal to $x_{j} / \varphi$. In the absence of input-output linkages, $\gamma=1$, the input cost index $x_{j}$ boils down to the wage rate $w_{j}$.

Turning to profit maximizing output levels, we must now specify policy and trade cost wedges. Government policies introduce price wedges, and real trade costs give rise to quantity wedges. Thus, if $t_{j i}$ is the ad valorem tax (subsidy if negative) on the use (sale) of a good originating in country $j$ and sold in country $i$, and if $\tau_{j i} \geq 1$ is the iceberg-type real trade cost caused by shipping a good from $j$ to $i$, then we have

$$
\begin{equation*}
p_{j i}(\varphi)=\frac{\tilde{p}_{j i}(\varphi)}{1+t_{j i}} \quad \text { and } \quad q_{j i}(\varphi)=\tau_{j i} \tilde{q}_{j i}(\varphi), \tag{11}
\end{equation*}
$$

where $p_{j i}(\varphi)$ is the net of tax (ex factory) price of a good produced by firm $\varphi$ located in $j$ and sold in $i$, and $q_{j i}(\varphi)$ is the quantity that firm $\varphi$ in country $j$ has to produce in order to deliver $\tilde{q}_{j i}(\varphi)$ units of its variety in country $i$.

As usual, we assume that firms take the prices of final goods, $\tilde{P}_{j}$, as given for all $j$. Moreover, we assume market segmentation, whence profit maximizing prices can be determined independently for all destinations $i$. The profit maximizing problem solved by firm $\varphi$ in country $j$ therefore is

$$
\begin{equation*}
\max _{p_{j i}(\varphi) \geq 0}\left\{p_{j i}(\varphi) \tilde{q}_{j i}(\varphi)-\frac{x_{j}}{\varphi} q_{j i}(\varphi)-w_{j} f_{j i}\right\}, \tag{12}
\end{equation*}
$$

where $f_{j i}$ denotes the fixed cost that a firm located in country $j$ has to incur in order to serve consumers or producers in country $i$. We assume that these costs are the same for goods shipped for the purpose of final goods assembly and for the purposes of intermediate input use, and that they are independent of the firms productivity. Note also that final assembly demand is governed by (6) above. The first order condition for this maximization problem implies the following pricing rule for a firm with productivity $\varphi$ :

$$
\begin{equation*}
p_{j i}(\varphi)=\frac{\sigma}{\sigma-1} \frac{\tau_{j i} x_{j}}{\varphi} \tag{13}
\end{equation*}
$$

where $\frac{\sigma}{\sigma-1}$ is the mark-up of prices over marginal cost, inclusive of iceberg trade cost $\tau_{j i}$. Recall that consumers and producers in country $i$ face a price $\tilde{p}_{j i}(\varphi)=\left(1+t_{j i}\right) p_{j i}(\varphi)$, which includes a tax (subsidy) at rate $t_{j i}$ if $t_{i j}>0$ (if $t_{i j}<0$ ). Inserting (13) and (11) into conditional demand by country $i$ in (6), we obtain

$$
\begin{equation*}
\tilde{q}_{j i}(\varphi)=\left(\frac{\sigma}{\sigma-1} \frac{\tau_{j i} x_{j}}{\varphi}\right)^{-\sigma} \frac{Y_{i} \tilde{P}_{i}^{\sigma-1}}{\left(1+t_{j i}\right)^{\sigma}} . \tag{14}
\end{equation*}
$$

Note that iceberg trade costs and the consumption tax-cum-subsidy are completely symmetric in their effect on demand. However, production $q_{j i}(\varphi)=\tau_{j i} \tilde{q}_{j i}(\varphi)$, differs as iceberg trade costs require the use of resources, whereas commercial policy does not.

Net of tax revenue of a firm with productivity $\varphi$ located in $j$ from selling to $i$ is

$$
\begin{equation*}
r_{j i}(\varphi)=p_{j i}(\varphi) \tilde{q}_{j i}(\varphi)=\left(\frac{\sigma}{\sigma-1} \frac{\tau_{j i} x_{j}}{\varphi}\right)^{1-\sigma} \frac{Y_{i} \tilde{P}_{i}^{\sigma-1}}{\left(1+t_{j i}\right)^{\sigma}} . \tag{15}
\end{equation*}
$$

Maximum profits earned by firm $\varphi$ located in country $j$ from selling in country $i$, net of fixed costs of market access, are

$$
\begin{equation*}
\pi_{j i}(\varphi)=\frac{1}{\sigma-1}\left(\frac{\tau_{j i} x_{j} \tilde{q}_{j i}(\varphi)}{\varphi}-(\sigma-1) w_{j} f_{j i}\right) . \tag{16}
\end{equation*}
$$

### 2.4 Selection and entry

Firms first decide about entry, based on a fixed entry $\operatorname{cost} w_{j} f_{j}^{e}$ and expected productivity, and once they have entered they select themselves into different markets, based on fixed costs of market access $w_{j} f_{j i}$ and observed productivity. In the spirit of backward induction, we first look at the selection effect, which determines the equilibrium threshold levels of productivity $\varphi_{j i}^{*}$, and then turn to entry.

The presence of a fixed cost of access to national markets implies threshold levels of productivity that firms in any one country need to surpass in order to take up selling in domestic as well as foreign markets. Having learned about its productivity subsequent to entry, a firm will sell to a given market $i$ only if it earns a positive profit from doing so. The threshold productivity level for a firm in country $j$ to select itself into selling to market $i$ is denoted by $\varphi_{j i}^{*}$
and determined by the condition $\pi_{j i}\left(\varphi_{j i}^{*}\right)=0$. Solving this condition for $\varphi_{j i}^{*}$ yields

$$
\begin{equation*}
\varphi_{j i}^{*}=\frac{\sigma^{\frac{\sigma}{\sigma-1}}}{\sigma-1} f_{j i}^{\frac{1}{\sigma-1}} \tau_{j i}\left(1+t_{j i}\right) x_{j} w_{j}^{\frac{1}{\sigma-1}} \tilde{P}_{i}^{-1}\left(\frac{Y_{i}}{1+t_{j i}}\right)^{-\frac{1}{\sigma-1}} . \tag{17}
\end{equation*}
$$

This equation highlights a selection effect of trade costs and tax policy. A number of observations are worth pointing out. First, other things equal, a rise in real trade costs $\tau_{j i}$ increases the threshold level of productivity that separates country $j$ firms exporting to country $i$ from those that do not, on the same footing as does a rise in marginal cost $x_{j}$; selection into exporting becomes tougher. Secondly, an equiproportional increase in the fixed market access $\operatorname{cost} f_{j i}$ and the variable real trade $\operatorname{cost} \tau_{j i}$ affects the threshold $\varphi_{j i}^{*}$ in the same way as does an equiproportional increase in the tax wedge $1+t_{j i}$. And finally, the second line decomposes the effect of the tax policy into a direct substitution effect, which works on the same footing as with real trade costs, and an effective market size effect that works through deflating total expenditure by country $i$ by $\left(1+t_{j i}\right)^{-1}$. Clearly, being more expensive on account of $t_{j i}, \tau_{j i}$ or $x_{j}$ makes it more difficult for firms from $j$ to enter $i$, which in turn raises country $i$ 's price index. Moreover, other things equal, covering the fixed market access cost $f_{j i}$ is more difficult if the market is smaller (in terms of lower expenditure $Y_{i}$ ).

It must be emphasized, however, that these are partial equilibrium effects since they ignore general equilibrium repercussions captured by the endogenous variables $w_{j}, \tilde{P}_{i}$ and $Y_{i}$ on the right-hand side of (17). The presence of intermediate inputs, $\gamma<1$, affects these repercussions. For instance, it effectively waters down the effect that an increase in country $j$ 's wage rate, say in a scenario of falling real trade costs, has on the threshold level of exporting. The reduction in destination country $i$ 's price index that occurs in the general equilibrium adjustment of such a scenario similarly has a watered down effect on the export threshold level $\varphi_{j i}^{*}$. Since the two feedback effects work in opposite directions, the net effect of them being watered down by the presence of intermediate inputs is ambiguous, as regards the general equilibrium adjustment of $\varphi_{j i}^{*}$.

Free entry implies that expected profits from selling to all markets is equal to the entry
cost:

$$
\begin{equation*}
\sum_{i=1}^{M} \int_{\varphi_{j i}^{*}} \pi(\varphi) g(\varphi) \mathrm{d} \varphi=w_{j} f_{j}^{e} \tag{18}
\end{equation*}
$$

This is an equilibrium condition stating that a potential entrant expects zero profits, given the distribution of the productivity level as captured by the density $g(\varphi)$, with an associated distribution function $G(\varphi)$. Plausibly, and for the sake of a closed form solution for the integral, we assume a Pareto distribution for $\varphi$, with a shape parameter denoted by $\theta>0$. To guarantee a finite average productivity in equilibrium, we further assume $\theta>\sigma-1$. We show in the appendix that with this additional assumption the zero profit equilibrium condition may be written as

$$
\begin{equation*}
\sum_{i=1}^{M} f_{j i}\left(\varphi_{j i}^{*}\right)^{-\theta}=\frac{\theta-(\sigma-1)}{\sigma-1} f_{j}^{e} \tag{19}
\end{equation*}
$$

Although not directly evident from (19), the zero profit equilibrium condition is of key importance for the determination of $N_{j}$, the equilibrium mass of firms entering in order to take up production in country $j$. We assume that firms live for one period only, whence $N_{j}$ also denotes the number of potential producers in country $j$. However, all firms with $\varphi<\min _{i}\left\{\varphi_{j i}^{*}\right\}$ never start producing, although the entire mass of firms incur the entry cost $w_{j} f_{j i}$. The mass of firms located in country $j$ and serving country $i$ is given by

$$
\begin{equation*}
N_{j i}=N_{j}\left[1-G\left(\varphi_{j i}^{*}\right)\right]=N_{j}\left(\varphi_{j i}^{*}\right)^{-\theta}, \tag{20}
\end{equation*}
$$

where the second equality follows from the Pareto distribution with a shape parameter $\theta$.
It is instructive to see that entry-country's wage rate $w_{j}$ drops from the zero profit condition (19) since it appears both, in the equations describing the selection into markets, as appearing in (17), and on the right-hand side of (18). The underlying assumption here is that fixed costs of market entry in country $i$ draw on resources from the sending country $j$. Moreover, the free entry equilibrium condition (19) implies that a policy that affects one productivity threshold $\varphi_{i j}^{*}$ has repercussions on at least one other threshold $\varphi_{i k \neq j}^{*}$ in order to restore zero profits in expectations.

### 2.5 Goods market equilibrium

The remaining equilibrium conditions relate to goods and labor markets. Given goods market equilibrium for each variety produced by country labor market will be in equilibrium due to Walras' Law which implies balanced trade. We thus close our model by a balanced trade condition as well as a goods market equilibrium condition for each of our $M$ countries. It proves convenient to first pin down the price of the final good which is equal to the unit cost of final goods assembly given in (5). Using the markup pricing condition for firm $\varphi$ of country $j$ when selling to country $i$ as given in (11), it can be shown (see the appendix) that

$$
\begin{equation*}
\tilde{P}_{i}=\left(\sum_{j=1}^{M} N_{j} \chi_{j i} \xi_{j i}\right)^{\frac{1}{1-\sigma}}, \tag{21}
\end{equation*}
$$

where $\chi_{j i}:=\left(\frac{\sigma}{\sigma-1}\left(1+t_{j i}\right) \tau_{j i} x_{j}\right)^{1-\sigma}$ is an inverse measure of the freeness of exports from country $j$ to country $i$. It represents the intensive margin component in the minimum cost of assembly in country $i$ that would be present also without firm-heterogeneity. In turn, $\xi_{j i}:=\int_{\varphi_{j i}^{*}} \varphi^{\sigma-1} g(\varphi) \mathrm{d} \varphi$ represents the extensive margin component that derives from selection of firms into different markets. Assuming Pareto for $g(\varphi)$ implies $\xi_{j i}=\frac{\theta}{\theta-(\sigma-1)}\left(\varphi_{j i}^{*}\right)^{\sigma-\theta-1}$; see (54) in the appendix. ${ }^{14}$ As regards the selection effect, we have emphasized above that it involves two channels, the freeness of trade channel and the market size channel; see (17) and the subsequent discussion above. And finally, $N_{j}$ in (21) represents the extensive margin component that derives from firm entry.

It can be shown (see again the appendix) that the value of country $i$ expenditure falling on goods from country $j$, evaluated at country $i$ 's domestic prices, is equal to $N_{j} \chi_{j i} \xi_{j i} \times Y_{i} \tilde{P}_{i}^{\sigma-1}$. Denoting the share of country $j$ goods in country $i$ 's expenditure by $\lambda_{j i}$, we have

$$
\begin{equation*}
\lambda_{j i}=\frac{N_{j} \chi_{j i} \xi_{j i}}{\tilde{P}_{i}^{1-\sigma}} \tag{22}
\end{equation*}
$$

The close relationship between the price index $\tilde{P}_{i}$ and the expenditure shares $\lambda_{j i}$ is helpful

[^8]in that it will eventually allow us to express a country's aggregate welfare as a function of the share of expenditure on domestic goods. ${ }^{15}$

Demand for the good produced by firm $\varphi$ located in country $j$ is equal to $\sum_{i=1}^{M} q_{j i}(\varphi)$, where $q_{j i}(\varphi)=\tau_{j i} \tilde{q}_{j i}(\varphi)$ is given from (14) above. Equilibrium requires that $\sum_{i=1}^{M} q_{j i}(\varphi)=$ $q_{j}(\varphi)$, where $q_{j}(\varphi)$ denotes gross output of this firm, inclusive of (iceberg) real trade costs. This condition may be written as

$$
\begin{equation*}
q_{j}(\varphi)=\left[C_{j j}(\varphi)+C_{j} \cdot(\varphi)+M_{j j}(\varphi)+M_{j} \cdot(\varphi)\right] \tag{23}
\end{equation*}
$$

where $C_{j j}(\varphi)$ is final consumption demand originating from country $j$ 's own domestic households, while $C_{j} .(\varphi)$ is consumption demand originating from foreign countries' final consumption. Similarly, $M_{j j}(\varphi)$ denotes demand for intermediate input use by domestic firms, and $M_{j .}(\varphi)$ is demand for intermediate input use in other countries. Consumption demands are governed by the demand function as given in (14), with $Y_{i}$ being replaced by $w_{i} L_{i}+T_{i}$. In turn, intermediate input demands are governed by this same demand function (14), with $Y_{i}$ being replaced by the value foreign firms' demand for the bundle of intermediate inputs, in line with the demand function for $m_{i}(\varphi)$ as given in (9) above:

$$
\begin{align*}
C_{j j}(\varphi) & =\left(\frac{\sigma}{\sigma-1} \frac{x_{j}}{\varphi}\right)^{-\sigma} \frac{\tilde{P}_{j}^{\sigma}}{\left(1+t_{j j}\right)^{\sigma}} \times \frac{\left(w_{j} L_{j}+T_{j}\right)}{\tilde{P}_{j}}  \tag{24}\\
C_{j \cdot}(\varphi) & =\sum_{i \neq j}^{M} \tau_{j i}\left(\frac{\sigma}{\sigma-1} \frac{\tau_{j i} x_{j}}{\varphi}\right)^{-\sigma} \frac{\tilde{P}_{i}^{\sigma}}{\left(1+t_{j i}\right)^{\sigma}} \times \frac{\left(w_{i} L_{i}+T_{i}\right)}{\tilde{P}_{i}}  \tag{25}\\
M_{j j}(\varphi) & =\left(\frac{\sigma}{\sigma-1} \frac{x_{j}}{\varphi}\right)^{-\sigma} \frac{\tilde{P}_{j}^{\sigma}}{\left(1+t_{j j}\right)^{\sigma}} \times \frac{N_{j}}{\tilde{P}_{j}} \int_{\varphi_{j j}^{*}} m_{j}\left(\varphi_{j}\right) \tilde{P}_{j} g(\varphi) \mathrm{d} \varphi  \tag{26}\\
M_{j \cdot}(\varphi) & =\sum_{i \neq j}^{M} \tau_{j i}\left(\frac{\sigma}{\sigma-1} \frac{\tau_{j i} x_{j}}{\varphi}\right)^{-\sigma} \frac{\tilde{P}_{i}^{\sigma}}{\left(1+t_{j i}\right)^{\sigma}} \times \frac{N_{i}}{\tilde{P}_{i}} \int_{\varphi_{i i}^{*}} m_{i}\left(\varphi_{i}\right) \tilde{P}_{i} g(\varphi) \mathrm{d} \varphi \tag{27}
\end{align*}
$$

Note that demand is "factory gate demand" which is gross of the "iceberg cost" that will melt down on the way to a good's final delivery. Moreover, note that in each case demand has two factors, the first term capturing allocation of some category of aggregate demand to firm $\varphi$ in

[^9]country $j$, and the second term specifying the type of aggregate demand considered. Notice that these equations look at quantities, not values. Aggregate intermediate input demand in any one country is demand by all firms in existence, whence we must aggregate over all firms, noting that the least productive firm in any country has productivity $\varphi_{i i}^{*}$. Aggregation is possible in quantity terms, since aggregate demand by different firms is demand for the same type of aggregate good.

Aggregating (23) over domestic firms is possible only in value terms, since firms are producing differentiated goods. Consistently with the above, we evaluate total production by the firm's producer, or "factory gate" price, which is $p_{j j} .^{16}$

$$
\begin{equation*}
N_{j} \int_{\varphi_{j j}^{*}} p_{j j}(\varphi) q_{j}(\varphi) g(\varphi) \mathrm{d} \varphi=N_{j} \int_{\varphi_{j j}^{*}} p_{j j}\left[C_{j j}(\varphi)+C_{j} \cdot(\varphi)+M_{j j}(\varphi)+M_{j}\right][(\varphi) g(\varphi) \mathrm{d} \varphi . \tag{28}
\end{equation*}
$$

In the following, we use $Z_{j}:=N_{j} \int_{\varphi_{j j}^{*}} p_{j j}(\varphi) q_{j}(\varphi) g(\varphi) \mathrm{d} \varphi$ to denote aggregate output of country $j$, or total revenue of all firms located in $j$, evaluated at country $j$ 's producer prices $p_{j j}(\varphi)$. Of course, the value of output $Z_{j}$ is linked to total expenditure $Y_{j}$. Consumption expenditure derives from household income, which includes labor income plus government revenue $T_{j}$, assumed to be redistributed in lump-sum fashion; see (2). Note that $T_{j}$ can be negative (subsidy bill).

Cobb-Douglas technology implies that intermediate inputs command a share $1-\gamma$ of variable costs. In equilibrium, variable costs are a fraction $\frac{\sigma-1}{\sigma}$ of total revenue; see (13). The goods market equilibrium condition (28) may therefore be written as

$$
\begin{align*}
Z_{j}= & \frac{\lambda_{j j}}{1+t_{j j}}\left[\left(w_{j} L_{j}+T_{j}\right)+(1-\gamma) \frac{\sigma-1}{\sigma} Z_{j}\right] \\
& +\sum_{i \neq j} \frac{\lambda_{j i}}{1+t_{j i}}\left(w_{i} L_{i}+T_{i}\right) \\
& +\sum_{i \neq j} \frac{\lambda_{j i}}{1+t_{j i}}(1-\gamma) \frac{\sigma-1}{\sigma} Z_{i} \tag{29}
\end{align*}
$$

The right-hand side of this equation can be simplified by using $w_{j} L_{j}+T_{j}+(1-\gamma) \frac{\sigma-1}{\sigma} Z_{j}=Y_{j}$,

[^10]which leads to
\[

$$
\begin{equation*}
Z_{j}=\frac{\lambda_{j j}}{1+t_{j j}} Y_{j}+\sum_{i \neq j} \frac{\lambda_{j i}}{1+t_{j i}} Y_{i} \tag{30}
\end{equation*}
$$

\]

This states that gross domestic output is equal to domestic demand for this output plus exports. Balanced trade requires equal values for exports and imports at border prices, i.e.,

$$
\begin{equation*}
\sum_{i \neq j} \frac{\lambda_{j i}}{1+t_{j i}} Y_{i}=\sum_{i \neq j} \frac{\lambda_{i j}}{1+t_{i j}} Y_{j} \tag{31}
\end{equation*}
$$

Observing balanced trade, the goods market equilibrium may therefore be written as

$$
\begin{equation*}
Z_{j}=Y_{j} \sum_{i} \frac{\lambda_{i j}}{1+t_{i j}} \tag{32}
\end{equation*}
$$

Next, we explore the link between the value of production, $Z_{j}$, and value added. Recall that Cobb-Douglas technology implies that intermediate inputs take up a share $1-\gamma$ of variable cost, and that, in turn, variable cost are a fraction $\frac{\sigma-1}{\sigma}$ of $Z_{j}$. The zero profit condition plus labor market equilibrium then lead to

$$
\begin{equation*}
Z_{j}=\kappa \times w_{j} L_{j} \text {, where } \kappa:=\left[1-(1-\gamma) \frac{\sigma-1}{\sigma}\right]^{-1} \geq 1 \tag{33}
\end{equation*}
$$

This states that in value terms net output (i.e., net of domestic intermediate input use) must be equal to domestic value added, which is equal to households' net of tax labor income. We call the term $\kappa$ the "production value multiplier". It links value added $w_{j} L_{j}$ to the value of production $Z_{j}$. It incorporates input-output linkages and is - given our assumption of symmetry in the elasticity of substitution $\sigma$ and the labor cost share $\gamma$ - the same for all countries.

Combining this with the above goods market equilibrium condition (32) we obtain

$$
\begin{equation*}
w_{j} L_{j}+(1-\gamma) \frac{\sigma-1}{\sigma} Y_{j} \sum_{i} \frac{\lambda_{i j}}{1+t_{i j}}=\sum_{i} \frac{\lambda_{i j}}{1+t_{i j}} Y_{j} \tag{34}
\end{equation*}
$$

which may be rewritten as

$$
\begin{equation*}
Y_{j}=\tilde{\mu}_{j} \times w_{j} L_{j}, \quad \text { where } \tilde{\mu}_{j}:=\kappa\left(\sum_{i} \frac{\lambda_{i j}}{1+t_{i j}}\right)^{-1} \tag{35}
\end{equation*}
$$

We call the term $\tilde{\mu}_{j}$ the "gross output multiplier". It links value added $w_{j} L_{j}$ to the gross of tax value of output, and - broadly speaking - it incorporates the input output linkage as well as government tax policy. ${ }^{17}$ Notice that with uniform treatment of domestically produced and imported varieties $t_{j}=t_{i j}$ for all source countries $i$, the gross output multiplier simplifies to $\tilde{\mu}_{j}=\kappa\left(1+t_{j}\right)$, where the country-varying part is smaller (larger) than one if the country implements a subsidy (tax).

Looking at tax revenue in somewhat more detail, we have

$$
I_{j}=w_{j} L_{j}+T_{j}=w_{j} L_{j}+Y_{j} \sum_{i} \frac{t_{i j} \lambda_{i j}}{1+t_{i j}}
$$

In the appendix we show that inserting (35) yields

$$
\begin{equation*}
I_{j}=\mu_{j} \times w_{j} L_{j} \quad \text { where } \mu_{j}:=\tilde{\mu}_{j} \sum_{i}\left(\kappa^{-1}+t_{i j}\right) \frac{\lambda_{i j}}{1+t_{i j}} . \tag{36}
\end{equation*}
$$

We shall henceforth refer to $\mu_{j}$ as the "income multiplier"; it links value added to income, incorporating lump-sum redistribution (financing) of tax revenue (subsidy bill). Notice that in the absence of input-output linkages, $\gamma=1$, we have $\mu_{j}=\tilde{\mu}_{j}=\left(\sum_{i} \frac{\lambda_{i j}}{1+t_{i j}}\right)^{-1}$, which is intuitive.

### 2.6 Mass of entrants

We now use the equilibrium conditions derived above to solve for the mass of entrants $N_{i}$, before turning to welfare in the next subsection. Employing the definition of the various margins and the zero cutoff profit condition (17), we can write aggregate (net-of-tax) sales of firms from country $i$ in country $j$ as (see appendix for details)

$$
\begin{equation*}
\frac{\lambda_{i j}}{1+t_{i j}} Y_{j}=\frac{\theta \sigma}{\theta-(\sigma-1)} N_{i}\left(\varphi_{i j}^{*}\right)^{-\theta} w_{i} f_{i j} . \tag{37}
\end{equation*}
$$

[^11]This relationship tells that aggregate sales can be written as the product of the sales of the average firm and the mass of firms located in $i$ active in $j$.

Using the free entry condition (19) to substitute out expected sales and solving for $N_{i}$, we obtain

$$
N_{i}=\frac{\sigma-1}{\theta \sigma w_{i} f_{i}^{e}} \sum_{j} \frac{\lambda_{i j}}{1+t_{i j}} Y_{j}=\frac{\sigma-1}{\theta \sigma f_{i}^{e}} \frac{Z_{i}}{w_{i}},
$$

where the equality follows from (32). As in the standard case, the equation commands that the mass of entrants is proportional to the ratio of the value of production $Z_{i}$ and the wage rate $w_{i}$. While in the absence of input-output linkages free entry implies that all the production value is used to finance the wage bill, in their presence firms also have to finance for material input, which drives a wedge between the production value and value added (labor compensation). Using the zero profit condition (33) to substitute out $Z_{i}$, the equilibrium mass of entrants is given by

$$
\begin{equation*}
N_{i}=\frac{\sigma-1}{\theta \sigma f_{i}^{e}} \kappa L_{i} . \tag{38}
\end{equation*}
$$

In the presence of input-output linkages $(\gamma<1)$, the mass of entrants is larger than in their absence $(\gamma=1)$. The intuition is that for a given wage rate, the value of production must be larger with input-output linkages than without to finance material input.

Equation (38) implies that a constant fraction of workers is devoted to entry activity, independently of commercial policy. Key drivers of this invariance result are that net government revenue is fully rebated to workers and that there is only a single sector; see Caliendo et al. (2015). It will turn out below that the optimal policy is a subsidy, which has to be financed by a lump-sum tax on labor income. We therefore ignore cases without full rebate (or financing).

### 2.7 Welfare

Having characterized industry equilibrium, we are now ready to characterize welfare. Real income of the representative agent in country $i$ is given by $W_{i}=\mu_{i} \times w_{i} / \tilde{P}_{i}$, where $\mu_{i}$ is the income multiplier as defined in (36). In order to make the welfare formula comparable to the one popularized by Arkolakis et al. (2012), we need a mapping between the real wage and the domestic expenditure share.

Plugging in the definitions of the various margins into (22) and employing the zero cutoff profit condition (17) as well as the definition of the gross output multiplier (35), the domestic expenditure share can be written as

$$
\begin{equation*}
\lambda_{i i}=\frac{\theta\left(\sigma f_{i i} / L_{i}\right)^{1-\frac{\theta}{\sigma-1}}}{\theta-(\sigma-1)}\left(\frac{\sigma}{\sigma-1}\right)^{-\theta} \times N_{i} \tilde{\mu}_{i}^{\frac{\theta}{\sigma-1}-1}\left(1+t_{i i}\right)^{1-\frac{\sigma \theta}{\sigma-1}}\left(\frac{x_{i}}{\tilde{P}_{i}}\right)^{-\theta} . \tag{39}
\end{equation*}
$$

In a standard setting without input-output linkages and commercial policies, the above relationship simplifies to $\lambda_{i i} \propto\left(w_{i} / \tilde{P}_{i}\right)^{-\theta}$. The presence of commercial policy gives rise to (i) a direct effect comprising changes along the intensive margin and the extensive margin component stemming from selection for given aggregate variables, and (ii) a general equilibrium effect representing the change in the ease of fixed market access cost materialization $Y_{i} /\left(w_{i} f_{i i}\right)$ captured by the output multiplier. ${ }^{18}$ Input-output linkages interfere with the output multiplier effect. Moreover, they complicate the analysis as the input cost index does not simply reflect the wage rate.

Using (10) to substitute out $x_{i}$ from (39), we can solve for the real wage as

$$
\begin{align*}
\frac{w_{i}}{\tilde{P}_{i}} & =\zeta_{i}\left(N_{i} \times \tilde{\mu}_{i}^{\frac{\theta}{\sigma-1}-1} \times\left(1+t_{i i}\right)^{1-\frac{\sigma \theta}{\sigma-1}} \times \lambda_{i i}^{-1}\right)^{\frac{1}{\gamma \theta}}, \quad \text { where }  \tag{40}\\
\zeta_{i} & :=\left(\frac{\theta\left(L_{i} /\left(\sigma f_{i i}\right)\right)^{\frac{\theta}{\sigma-1}-1}}{\theta-(\sigma-1)}\left(\frac{\sigma-1}{\sigma}\right)^{\theta} A^{-\theta}\right)^{\frac{1}{\gamma \theta}} \tag{41}
\end{align*}
$$

collects parameters of the model. Intuitively, the presence of input-output linkages magnifies the effects of changes in the domestic expenditure share on the real wage in the absence of commercial policy.

Welfare formula. Combining the previous expressions, we can express welfare as

$$
\begin{equation*}
W_{i}=\zeta_{i} \times \mu_{i} \times\left(\tilde{\mu}_{i}^{1-\frac{\theta}{\sigma-1}} \times\left(1+t_{i i}\right)^{\frac{\sigma \theta}{\sigma-1}-1} \times \lambda_{i i}\right)^{-\frac{1}{\gamma \theta}} . \tag{42}
\end{equation*}
$$

[^12]This welfare formula generalizes the welfare formula presented in Arkolakis et al straightforwardly. Note that the effect of commercial policy on welfare through the domestic expenditure share bears the elasticity $1 /(\gamma \theta)$. For an ex post evaluation of welfare consequences of foreign policies and domestic trade policy, one additionally has to observe changes in the income multiplier and the gross output multiplier. Moreover, one needs information about an additional parameter, namely the elasticity of trade with respect to fixed costs $\frac{\theta}{\sigma-1}-1$, which governs the welfare consequences of a change in the gross output multiplier. ${ }^{19}$ For the ex post evaluation of domestic commercial policy, one also has to back out the elasticity of trade flows in domestic commercial policy, $\frac{\sigma \theta}{\sigma-1}-1$.

Domestic expenditure share. In order to characterize optimal policies, we have to rely on the ex ante evaluation of welfare consequences of commercial policy, which involves the effect of commercial policy on welfare through changes in the domestic expenditure share. In order to pave the ground for this type of analysis, we derive an expression for the domestic expenditure share. Starting from the expression for the price index, we show in the appendix that the domestic expenditure share can be written as

$$
\begin{equation*}
\lambda_{i i}=\left(1+\sum_{j=1}^{M} \frac{N_{j}}{N_{i}} \tau_{j i}^{-\theta}\left(\frac{1+t_{j i}}{1+t_{i i}}\right)^{1-\frac{\sigma \theta}{\sigma-1}}\left(\frac{f_{j i}}{f_{i i}}\right)^{\frac{\sigma-1-\theta}{\sigma-1}}\left(\frac{w_{j}}{w_{i}}\right)^{\frac{\sigma-1-\theta}{\sigma-1}}\left(\frac{x_{j}}{x_{i}}\right)^{-\theta}\right)^{-1} . \tag{43}
\end{equation*}
$$

This expression highlights that with uniform policies $t_{i i}=t_{j i}$ for all source countries $j$, commercial policies have a general equilibrium effect on domestic expenditure shares through relatives wage and relative input cost indices.

Employing the definition of the input cost index and using equation (40) to substitute the real wage, the domestic expenditure share emerges as

$$
\begin{align*}
\lambda_{i i} & =\left(1+\sum_{j=1}^{M} \frac{N_{j}}{N_{i}} \tau_{j i}^{-\theta}\left(\frac{1+t_{j i}}{1+t_{i i}}\right)^{1-\frac{\sigma \theta}{\sigma-1}}\left(\frac{f_{j i}}{f_{i i}}\right)^{\frac{\sigma-1-\theta}{\sigma-1}}\left(\frac{w_{j}}{w_{i}}\right)^{1-\frac{\theta \sigma}{\sigma-1}} \Gamma_{i j}\right)^{-1},  \tag{44}\\
\Gamma_{i j} & :=\left(\left(\frac{\xi_{j}}{\xi_{i}}\right)^{\theta \gamma} \frac{N_{j}}{N_{i}}\left(\frac{\tilde{\mu}_{j}}{\tilde{\mu}_{i}}\right)^{\frac{\theta}{\sigma-1}-1}\left(\frac{1+t_{j j}}{1+t_{i i}}\right)^{1-\frac{\sigma \theta}{\sigma-1}}\left(\frac{\lambda_{j j}}{\lambda_{i i}}\right)^{-1}\right)^{\frac{1-\gamma}{\gamma}} .
\end{align*}
$$

[^13]This expression highlights the implications of input-output linkages for domestic expenditure shares. In the absence of input-output linkages ( $\gamma=1$ ), we have $\Gamma_{i j}=1$, and domestic expenditure shares are a function of relative wages and wedges between trade and domestic policies of countries. With input-output linkages, they additionally depends on relative domestic expenditure shares, relative gross output multipliers, and relative domestic policy wedges.

## 3 Cooperative commercial policies

In this section, we consider the case where potentially asymmetric countries cooperatively determine their commercial policies in order maximize their joint welfare. This perspective prevents countries from conducting beggar-thy-neighbor policies. Hence, the scenario analyzed in this section is free of terms-of-trade considerations, which allows us to discuss the optimal policy implications of the input distortion inherent to a monopolistic competition model of international trade with input-output linkages. We explore the interaction of the input distortion with the terms-of-trade externality in the next section.

In the first subsection, we shall assume that domestically produced and imported inputs are treated uniformly by policy makers, restricting the policy choices to one per country. This greatly simplifies the analysis for the following reasons. Firstly, not discriminating against imported intermediate goods allows to determine income and gross output multipliers independently of the domestic expenditure share. Secondly, the domestic expenditure share, which is itself an determinant of welfare (see welfare formula (42)), is not directly driven by commercial policy.

With symmetric countries, domestic expenditure shares do not respond to changes in commercial policy. Policy makers trade off changes in the income multiplier as well as the gross output multiplier and the direct effect of commercial policy, where the last two terms emerge when we condition on the domestic expenditure share. With asymmetric countries, however, commercial policy potentially affect domestic expenditure shares through the general equilibrium adjustments of wages and input cost indices. It turns out that expenditure
shares are not affected,
In the second subsection, we turn to the case where governments only have one instrument at their disposal, referring to situations where either trade policy interventions are restricted, e.g., by international trade agreements, or domestic policy interventions are unwelcome for some reasons. In order to pave the ground for the analysis of optimal noncooperative trade policy, which we take up in the next section, we focus on the latter. ${ }^{20} \mathrm{We}$ characterize optimal trade policy addressing the input distortion in the absence of terms-oftrade considerations.

### 3.1 Cooperative uniform policies

With uniform treatment of domestically and imported intermediate goods, i.e., $t_{j i}=t_{i i}$ for all source countries $j$, income and gross output multipliers emerge as

$$
\begin{equation*}
\mu_{i} \equiv 1+\kappa t_{i} \text { and } \tilde{\mu}_{i}=\kappa\left(1+t_{i}\right), \tag{46}
\end{equation*}
$$

where $\kappa:=\left[1-(1-\gamma) \frac{\sigma-1}{\sigma}\right]^{-1}$ is the product value multiplier; see (33). Notice that the multipliers are determined independently of expenditure shares. The reason is that with uniform treatment of domestically produced and imported goods, we can factor out policy wedges in the summation over policy-adjusted expenditure shares and exploit that expenditure shares add up to unity.

Expression (46) implies that the direct effect of commercial policy on welfare and its effect through the gross output multiplier can be combined by a new variable $\Omega_{i}\left(t_{i}\right)$ defined as

$$
\begin{equation*}
\Omega_{i}\left(t_{i}\right):=\left[\tilde{\mu}_{i}\left(t_{i}\right)^{1-\frac{\theta}{\sigma-1}} \times\left(1+t_{i}\right)^{\frac{\sigma \theta}{\sigma-1}-1}\right]^{-\frac{1}{\gamma \theta}} \propto\left(1+t_{i}\right)^{-\frac{1}{\gamma}} . \tag{47}
\end{equation*}
$$

In a closed-economy version of the model $(\lambda=1), \Omega_{i}\left(t_{i}\right)$ represents the effect of commercial policy on welfare through the real wage $w / \tilde{P}$. We will see below that this intuition also car-

[^14]ries over to the case of potentially asymmetric open economies, where domestic expenditure shares turn out to be determined independently of the stance of commercial policies, if all countries implement the same policy.

Interestingly, the elasticity of $\Omega_{i}\left(t_{i}\right)$ in $1+t_{i}$ only depends on the labor cost share $\gamma$, but is independent of the measure of firm heterogeneity $\theta$ and the elasticity of substitution. This observation implies that with uniform treatment of domestically produced and imported varieties, commercial policy only affects, on net, the price of the composite good via the intensive margin. The term $\gamma$ reflects the "loop" generated by taking into account that the price index is a function of the input cost index, which, is turn, is a function of the price index.

Equation (47) implies that a uniform subsidy raises the real wage. This is intuitive, as a subsidy lowers the price of all varieties bundled in the composite good. Notice that this observation does not necessarily means that a subsidy is welfare enhancing as the welfare calculus has to take into account that the subsidy must be financed by means of a lump-sum tax on labor income.

Symmetric countries. With symmetric countries, uniform commercial policy has no bearing on the domestic expenditure share; see equation (43). The joint welfare maximization problem can be stated as

$$
\max _{t} \tilde{W}=\mu(t) \times \Omega(t)
$$

see welfare formula (42) and (47).
The optimal policy intervention $t^{*}$ is obtained by setting $\partial \tilde{W} / \partial t=0$ :

$$
\frac{\partial \tilde{W}\left(t^{*}\right)}{\partial t}=\Omega\left(t^{*}\right) \frac{\partial \mu\left(t^{*}\right)}{\partial t}+\mu\left(t^{*}\right) \frac{\partial \Omega\left(t^{*}\right)}{\partial t} \stackrel{!}{=} 0 .
$$

Notice that in the absence of input-output linkages $(\gamma=1)$, the objective function $\tilde{W}$ is invariant to changes in commercial policy; compare equations (46) and (47). Hence, the effect of commercial policy via the real wage $\Omega(t)$ is always exactly offset by changes in the income multiplier.

In the presence of input-output linkages, the first-order condition of the welfare-maximization
problem implies ${ }^{21}$

$$
\begin{equation*}
1+t^{*}=\frac{1-\kappa^{-1}}{1-\gamma}=\frac{\sigma-1}{\sigma} \tag{48}
\end{equation*}
$$

This result highlights that the effect of a subsidy on welfare through changes in the price index outweigh its effect through the income multiplier, if the subsidy is not too large. The optimal policy that maximizes joint welfare of symmetric countries is a subsidy of rate $\left|t^{*}\right|=$ $1 / \sigma$. In contrast to the standard Melitz (2003) case without input-output linkages, laissezfaire market outcomes are not socially optimal. ${ }^{22}$

The optimal policy exactly offsets the markup over marginal cost each intermediate good producer, which directly feeds into the price index (5) via the term $\chi$ representing the intensive margin. The "correction" of the price of a good clearly has repercussions on demand for that good and on firms' revenue and profits. While firms' profits are in general decisive in the determination of cutoff productivity levels, we show in the appendix that with symmetric countries and uniform treatment of domestically produced and imported goods, commercial policy has no effect on selection. Recall from section 2 that commercial policy has no bearing on the extensive margin component that works through firm entry either; see equation (38). As in our setting all firms charge the same markup, firm heterogeneity has no optimal policy implications. Moreover, the labor cost share $\gamma$ does not interfere with the markup. Therefore, the optimal policy is also independent of the labor cost share $\gamma$.

Optimal commercial policy raises the overall efficiency of the economy by correcting an input distortion. In the presence of markups, the price of the composite good is too high such that intermediate good producers use too little material input into production; see the conditional input demands (9). By using more of the composite good for production of inputs, each intermediate good producers produces more efficiently, which translates into an aggregate productivity gain. Hence, in our setting efficiency gains do nor arise from reallocation of resources across firms, but from a more efficient allocation of the composite good to final

[^15]consumption and intermediate input use.

Illustrative example. In order to illustrate that the policy-driven decentralized equilibrium implements the planner solution, consider the case of a closed economy with a fixed mass of $N$ symmetric firms. ${ }^{23}$ Let $q$ denote output per firm.Aggregate output is given by $Q=$ $\left(N \times q^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}=N^{\frac{\sigma}{\sigma-1}} \times q$.

Quantity $C$ of the aggregate good is for consumption, whereas quantity $N \times m$ is used as input into production, where $m$ denotes input per firm. Output per firm is given by $q=$ $l^{\gamma} m^{1-\gamma}$, where the productivity level of the firm is normalized to unity. Labor input per firm $l$ is a constant fraction of labor total endowment. ${ }^{24}$

The planner chooses $m$ in order to maximize real consumption $C$ subject to the production technologies $Q$ and $q$ :

$$
\max _{m} C=Q-N m=N^{\frac{\sigma}{\sigma-1}} l^{\gamma} m^{1-\gamma}-N m .
$$

The first-order condition of the real consumption maximization problem implies that relative demand for workers emerges as

$$
\frac{l}{m}=(1-\gamma)^{-\frac{1}{\gamma}} N^{-\frac{1}{\gamma(\sigma-1)}} .
$$

In the policy-driven decentralized equilibrium, relative input demand is $l / m=\gamma /[(1-\gamma) \Omega]$, where the real wage is given by $\Omega=\left(N^{-\frac{1}{\sigma-1}} \times \frac{\sigma}{\sigma-1} \times A \times(1+t)\right)^{-\frac{1}{\gamma}} .{ }^{25}$ Hence, relative demand for workers is

$$
\frac{l}{m}=(1-\gamma)^{-\frac{1}{\gamma}} \times N^{-\frac{1}{\gamma(\sigma-1)}} \times\left(\frac{\sigma}{\sigma-1} \times(1+t)\right)^{\frac{1}{\gamma}}
$$

[^16]Comparing this expression to the one obtained from the planner problem, we find that with the optimal subsidy $1+t^{*}=\frac{\sigma-1}{\sigma}$, the policy-driven decentralized equilibrium exactly resembles the planner outcome. Hence, starting from the laissez-faire equilibrium, the efficiency of the economy can be increased by allocating more of the composite to input producers.

Asymmetric countries. With asymmetric countries, we have to back out the effect of commercial policy on domestic expenditure shares and relative wages. With uniform treatment, commercial policies do not exhibit direct effects on domestic expenditure shares, but potentially affect them through general equilibrium adjustments in the relative wage and the $\Gamma_{i j}$ terms:

$$
\begin{align*}
\lambda_{i i} & =\left(1+\sum_{j=1}^{M} \frac{N_{j}}{N_{i}} \tau_{j i}^{-\theta}\left(\frac{f_{j i}}{f_{i i}}\right)^{\frac{\sigma-1-\theta}{\sigma-1}}\left(\frac{w_{j}}{w_{i}}\right)^{1-\frac{\theta \sigma}{\sigma-1}} \Gamma_{i j}\right)^{-1}, \quad \text { where }  \tag{49}\\
\Gamma_{i j} & =\left(\left(\frac{\xi_{j}}{\xi_{i}}\right)^{\theta \gamma} \frac{N_{j}}{N_{i}}\left(\frac{1+t_{j}}{1+t_{i}}\right)^{\theta}\left(\frac{\lambda_{j j}}{\lambda_{i i}}\right)^{-1}\right)^{\frac{1-\gamma}{\gamma}} . \tag{50}
\end{align*}
$$

With uniform treatment, the $\Gamma_{i j}$ terms collapse to functions of relative domestic expenditure shares and relative policies.

Relative wages are determined by the balanced trade conditions

$$
\sum_{i \neq j} \frac{\lambda_{j i}}{1+t_{i}} \tilde{\mu}_{i} w_{i} L_{i}=\frac{\tilde{\mu}_{j} w_{j} L_{j}}{1+t_{j}} \sum_{i \neq j} \lambda_{i j}=\frac{\tilde{\mu}_{j} w_{j} L_{j}}{1+t_{j}}\left(1-\lambda_{j j}\right),
$$

where we have used $Y_{i}=\tilde{\mu}_{i} w_{i} L_{i}$ to replace gross outputs and $\sum_{i} \lambda_{i j}=1$. Uniform treatment implies that balanced trade can be rewritten as

$$
\begin{equation*}
\sum_{i \neq j} \lambda_{j i} w_{i} L_{i}=w_{j} L_{j}\left(1-\lambda_{j j}\right) \tag{51}
\end{equation*}
$$

where we have used equation (46).
The joint welfare maximization problem of $M$ potentially asymmetric countries reads

$$
\max _{\left\{t_{j}\right\}} W \equiv \sum_{j} \zeta_{j} \times \mu_{j}\left(t_{j}\right) \times \tilde{\mu}_{j}\left(t_{j}\right)^{\frac{\theta-(\sigma-1)}{\gamma(\sigma-1)}} \times\left(1+t_{j}\right)^{-\frac{1}{\gamma \theta}\left(\frac{\sigma \theta}{\sigma-1}-1\right)} \times \lambda_{j j}(\mathbf{t})^{-\frac{1}{\gamma \theta}},
$$

where domestic expenditure shares are in general complicated functions of all countries' policies summarized in vector $t$.

Notice that the input distortion is not related to the share of domestic varieties in the composite good, as both domestically produced and imported varieties are subject to the same markup. Hence, from a joint welfare perspective, we are not interested in policies that affect expenditure shares. Inspection of the system of equations (49)-(51) shows that uniform commercial policies do not affect expenditure shares, if all countries implement the same policy $t$.

With this restriction, the joint welfare maximizing problem can be rewritten as

$$
\max _{t} W \equiv \sum_{j} \zeta_{j} \times \mu_{j}(t) \times \tilde{\mu}_{j}(t)^{\frac{\theta-(\sigma-1)}{\gamma \theta(\sigma-1)}} \times(1+t)^{-\frac{1}{\gamma \theta}\left(\frac{\sigma \theta}{\sigma-1}-1\right)} \times \lambda_{j j}^{-\frac{1}{\gamma \theta}} .
$$

The first-order condition of this joint welfare maximization problem mimics the one obtained in the symmetric country case. Hence, the optimal policy result described above for symmetric countries carries over to the case of asymmetric countries.

### 3.2 Efficiency gains of moving from laissez-faire to social optimum

Before turning to the case of optimal cooperative trade policy, we want to gauge the importance of optimal policy. In order to obtain clear-cut results, we analyze the welfare gains of moving from laissez-faire equilibrium to social optimum reached by the implementation of the first-best consumption subsidy for the closed economy case. Let $l f$ and * denote variables obtained under laissez-faire and optimal-policy driven equilibria, respectively. Moreover, let $\rho:=\frac{\sigma-1}{\sigma} \in(0,1)$ be an inverse measure of the mark-up. Then, the change in welfare induced by moving from laissez-faire to a policy-driven equilibrium is given by

$$
\frac{W^{*}}{W^{l f}}-1=\frac{\mu^{*} / \tilde{P}^{*}}{\mu^{l f} / \tilde{P}^{l f}}-1=\frac{\gamma \rho^{-\frac{1-\gamma}{\gamma}}}{1-(1-\gamma) \rho}-1,
$$

where we have used $\mu^{l f}=1$ and $\tilde{P}^{*} / \tilde{P}^{l f}=\rho^{\frac{1}{\gamma}}$.
As discussed above, in the absence of input-output linkages we have $W^{*} / W^{l f}=1$. In the
limiting case where markups vanish $(\sigma \rightarrow \infty \Rightarrow \rho \rightarrow 1)$, we have $W^{*} / W^{l f} \rightarrow 1$. We show in the appendix that an increase in the markup (lower $\rho$ ) magnifies the welfare gains of moving from laissez-faire to the optimal policy driven equilibrium. Intuitively, the gains from repairing the distortion are larger, the more severe the distortion. Moreover, we establish in the appendix that the welfare gains of moving from laissez-faire to social optimum are larger, the smaller $\gamma$. Clearly, the smaller $\gamma$, the more relevant is the input distortion.

Figure 1 graphically illustrates the welfare gains for different values of $\gamma$ and $\sigma$. We let the labor cost share vary between 0.1 and 1 and depict the welfare gains for $\sigma=5$ and $\sigma=$ 10, which are reasonable bounds in single-sector studies; see Anderson and van Wincoop (2003). The left panel shows that the welfare gains can be sizable, if input-output linkages are sufficiently important. In the right panel, we zoom in and take a closer look at cost shares in the interval $\gamma \in(0.7,1)$.

Figure 1: Welfare gains of moving from laissez-faire to policy-driven equilibrium


### 3.3 Cooperative trade policy

We now assume that the governments are restricted to set their trade policies cooperatively, while domestic market interventions are ruled out. ${ }^{26}$ For the sake of illustration, we focus on symmetric countries. Given symmetry, there is only a single policy instrument which is applied to imports. Let $t$ denote this symmetric trade policy.

[^17]The key difference to the scenarios considered above is that domestically produced goods are exempted from the policy. Commercial policy drives a wedge between prices of domestically produced and imported goods and therefore alters expenditure shares. This is an unwelcome side effect as changes in the expenditure shares are not required to address the input distortion, but rather constitute a deviation from the efficient spending allocation. In this sense, trade policy intervention is "costly".

In this setting, the multipliers can be written as

$$
\begin{equation*}
\mu=1+\kappa \frac{t(1-\lambda)}{1+\lambda t} \text { and } \tilde{\mu}=\kappa \frac{1+t}{1+\lambda t}, \tag{52}
\end{equation*}
$$

where we suppress country indices due to the symmetry assumption. Notice that, in contrast to the scenario above, the multipliers do depend on expenditure shares. The domestic expenditure share emerges as

$$
\lambda=\left(1+\eta(1+t)^{1-\frac{\sigma \theta}{\sigma-1}}\right)^{-1},
$$

where $\eta:=\tau^{-\theta}\left(\frac{f^{x}}{f^{d}}\right)^{1-\frac{\theta}{\sigma-1}}$ comprises variable trade costs and foreign over domestic market access costs $\left(f^{x} / f^{d}\right)$ and therefore can be viewed as a measure of the non-policy "freeness of trade".

The joint welfare maximization problem can be stated as

$$
\max _{t} \tilde{W}=\mu(t, \lambda(t)) \times \tilde{\mu}(t, \lambda(t))^{\frac{\theta-(\sigma-1)}{\gamma(\sigma-1)}} \times \lambda(t)^{-\frac{1}{\gamma \theta}} .
$$

We show in the appendix that

$$
\operatorname{sign} \frac{d W / W}{d t}=\operatorname{sign}\left[\frac{(1+t)\left(1-\left(\frac{\sigma \theta}{\sigma-1}-1\right) t \lambda\right)}{1+t-(\lambda t+1)(1-\gamma) \frac{\sigma-1}{\sigma}}-\frac{1}{\gamma}\left(1+\left(\frac{\sigma \theta}{\sigma-1}-1\right) \frac{t \lambda}{\sigma-1}\right)\right] .
$$

The optimal trade policy follows from setting this expression equal to zero and solving for $t$. The complication, however, is that the domestic expenditure share is itself a function of trade policy, which makes it impossible to come with closed-form solutions.

Figure 2: The labor cost share and optimal cooperative trade policy


Notes: We consider two symmetric countries. We set $\sigma=3.8$ and consider the limiting case $\theta=\sigma-1$. We assume that trade costs are absent, i.e. $\eta=1$. Under this parametrization, the optimal uniform policy would be $|t|=1 / \sigma \approx 0.26$, and a sufficient condition for $t>1 / \sigma$ is $\gamma>1 /(\sigma+1) \approx 0.21$.

We formally prove in the appendix by evaluating the welfare change at $t=0$ that the welfare-maximizing policy is an import subsidy. The optimal rate depends not only on the elasticity of substitution, but also on the labor cost share and the freeness of trade. ${ }^{27}$ It can be smaller or larger than the optimal cooperative uniform rate, depending on parameters.

The role of the labor cost share $\gamma$ for the optimal import subsidy is straightforward. In the absence of input-output linkages ( $\gamma=1$ ), there is no rationale for policy intervention. The input distortion is more severe, the smaller the labor cost share. Accordingly, the optimal import subsidy is larger (in absolute values), the smaller the labor cost share $\gamma$.

Figure 2 illustrates the optimal cooperative trade policy as a function of the labor cost share $\gamma$ for the two symmetric. We set $\sigma=3.8$ and consider the limiting case $\theta=\sigma-1$. We assume that trade costs are absent, i.e. $\eta=1$. The curve emerges from the point $(1,0)$, reflecting that laissez faire is optimal in the standard version of the Melitz (2003) without input-output linkages. The input distortion calls for an import subsidy, and (in absolute values) the rate is larger, the lower the labor-cost share $\gamma$. Under our parametrization, the import subsidy rate is (in absolute values) smaller than in the uniform treatment case $t=1 / \sigma \approx 0.26$ for $\gamma>0.20$. Notice that our sufficient condition requires $\gamma>1 /(\sigma+1) \approx 0.21$.

[^18]The role of the non-policy freeness of trade is more involved. It turns out that the optimal cooperative import subsidy is smaller, the smaller the freeness of trade, if the labor cost share is not too small. The intuition is the following. The ideal policy offsets the markup over marginal cost of all available varieties. Consider now the case where policy can only be applied to imported varieties. The policy can be used more efficiently, the larger the share of imports in the consumption basket, i.e., the higher the freeness of trade.

Figure 3 illustrates the role of freeness of trade for the optimal import subsidy for different labor cost shares. Notice that the optimal trade policy scale on the vertical axis in not the same in each panel. As above, we consider two symmetric countries, set $\sigma=3.8$ and $\theta=\sigma-1$. In the standard Melitz (2003) model, we have $\gamma=1$, and the optimal trade policy for symmetric countries would be laissez faire regardless of the level of trade costs (horizontal line at $t=0$ ). As argued above, with $\gamma<1$, an import subsidy is optimal. For sufficiently large values of $\gamma$, we find that (in absolute values) the import subsidy is increasing in the freeness of trade; see left panel. This observation reflects the fact that the import policy is less efficient in correcting for the input distortion, if the share of imported varieties is smaller. For small labor cost shares, the pattern can reverse, such that the optimal import subsidy is (in absolute values) decreasing in freeness of trade; see right panel. Note that For intermediate values of the labor cost share, we find a U-shaped relationship between $t$ and freeness of trade.

Figure 3: Freeness of trade and optimal cooperative trade policy


Notes: We consider two symmetric countries. We set $\sigma=3.8$ and consider the limiting case $\theta=\sigma-1$.

## 4 Non-cooperative trade policy

In this section, we consider policies that are optimal from a unilateral perspective, ignoring welfare effects on foreign economies. We focus on two countries, $H$ and $F$, assumed to be symmetric in all dimensions but commercial policies. In order to relate to the literature on optimal trade policy, we rule out domestic policy. The government sets the welfaremaximizing tax-cum-subsidy trade policy.

In the non-cooperative setting, the government does not take into account that imported varieties are subject to markups over foreign social (production) costs. Rather, the social costs of imported varieties are their prices at the border. From this perspective, in contrast to domestically produced goods, imported intermediate inputs are not too expensive and do not generate a problem in the cost-minimization problem of producers.

Another difference to the cooperative setting is that governments may follow beggar-thyneighbor strategies. In fact, the different perception of prices of domestically produced and imported intermediate inputs constitutes a rationale for the imposition of import tariffs or a subsidy on domestically produced goods - that exploit a terms-of-trade externality as in the standard Melitz case; see Demidova and Rodríguez-Clare (2009) and Felbermayr et al. (2013). ${ }^{28}$

In general equilibrium, an import tariff not only addresses the terms-of-trade externality, but also has repercussions on the input distortion. On impact, an import subsidy lowers the price index, which is welcome from the perspective of the input distortion. An import subsidy, however, cannot be the first-best instrument as the distortion arises from domestically produced intermediate goods. Trade policy can only be a vehicle to imperfectly address the input distortion. We aim at figuring out under which conditions one or the other rationale dominates, resulting in either optimal import tariffs or subsidies.

One implication immediately stands out. The optimal tariff must be smaller, if not even an import subsidy, than in the standard Melitz (2003) case without input-output linkages.

[^19]Moreover, the "freeness of trade" defined as $\eta:=\tau^{-\theta}\left(f^{x} / f^{d}\right)^{1-\frac{\theta}{\sigma-1}}$ comprising non-policy trade barriers, will be key as it drives the ratio of domestically produced and imported intermediate inputs. As can be seen from the analysis in section 3.3, optimal polices also hinge on the labor cost share $\gamma$ in the absence of uniform treatment of domestically produced and imported goods as well as on firm heterogeneity.

### 4.1 Preliminaries

In our setting with trade policy only, the multipliers are given by

$$
\begin{equation*}
\mu_{i}=1+\kappa \frac{t_{j i}\left(1-\lambda_{i i}\right)}{1+t_{j i} \lambda_{i i}} \text { and } \tilde{\mu}_{i}=\kappa \frac{1+t_{j i}}{1+t_{j i} \lambda_{i i}} . \tag{53}
\end{equation*}
$$

where $\kappa:=\left[\left(1-(1-\gamma) \frac{\sigma-1}{\sigma}\right)\right]^{-1}$ is the production value multiplier. Notice that the multipliers shown in (53) are essentially the same as in (53), the difference being that here we have to take into account cross country differences in trade policy and therefore domestic expenditure shares.

We simplify the analysis by assuming that only country $H$ conducts trade policy, while country $F$ abstracts from trade policy interventions. This assumption implies that $\tilde{\mu}_{F}=\kappa$ and $\mu_{F}=1$. Domestic expenditure shares are determined by equations (44) and (45). In the two-country setting, balanced trade can be written as

$$
\frac{w_{i}}{w_{j}}=\frac{1+t_{j i}}{1+t_{i j}} \frac{1-\lambda_{j j}}{1-\lambda_{i i}} \frac{\tilde{\mu}_{j}}{\tilde{\mu}_{i}} \frac{L_{j}}{L_{i}} .
$$

The optimal policy analysis requires to back out the effect of trade policy on the domestic expenditure share. We show in the appendix that

$$
\begin{aligned}
\frac{\frac{\mathrm{d} \lambda_{H H}}{\lambda_{H H}}}{\mathrm{~d} t_{F H}} & =\frac{\frac{1-\gamma}{\gamma}\left[1-\lambda_{F F}+\frac{\theta-(\sigma-1)}{\theta \sigma-(\sigma-1)} \frac{1-\lambda_{H H}}{t_{F H} \lambda_{H H}+1}\right]+\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F}+\frac{1-\lambda_{H H}}{t_{F H} \lambda_{H H}+1}}{1+\left(\frac{\theta \sigma}{\sigma-1}-1\right)\left(\lambda_{F F}+\frac{1+t_{F H}}{t_{F H} \lambda_{H H}+1} \lambda_{H H}\right)+\frac{1-\gamma}{\gamma}\left(2-\lambda_{H H}-\lambda_{F F}+\frac{\theta-(\sigma-1)}{\sigma-1} \frac{t_{F H} \lambda_{H H}\left(1-\lambda_{H H}\right)}{t_{F H} \lambda_{H H}+1}\right)} \\
& \times\left(1-\lambda_{H H}\right)\left(\frac{\theta \sigma}{\sigma-1}-1\right) \frac{1}{1+t_{F H}}>0,
\end{aligned}
$$

where the inequality follows from $\lambda_{H H}, \lambda_{F F}<1$ and $\theta>\sigma-1$. The expression highlights that

Figure 4: The role of the labor cost share

a tariff raises the domestic expenditure share.

### 4.2 Optimal trade policy

In the appendix, we derive the first-order condition of the welfare-maximization problem. Moreover, we show analytically that in the absence of trade cost $(\eta=1)$ and with an inactive selection effect $(\theta \rightarrow \sigma-1)$, the optimal policy is an import tariff if $\gamma>1 /(\sigma+1)$, while it is an import subsidy if $\gamma$ smaller than this threshold. This result implies that under the certain conditions, the input distortion dominates the terms-of-trade externality if the labor cost share is sufficiently small compared to the trade elasticity, which drives the terms-of-trade effect. Moreover, the result implies firm heterogeneity is not required for the input distortion to dominate the terms-of-trade externality.

Figure 4 draws on the first-order condition of the welfare maximization problem derived in the appendix and illustrates the welfare-maximizing trade policy as a function of the labor cost share $\gamma$. For $\gamma=1$, our analysis resembles the optimal tariff characterized in Gros (1987) and Felbermayr et al. (2013). For the given parametrization ( $\eta=1, \sigma=3.8, \theta=\sigma-1$ ), the critical $\gamma$ is approximately $\gamma \approx 0$.21.Qualitatively, Figure 4 resembles Figure 2, the difference being that here the terms-of-trade channel is active, such that the optimal policy is an import tariff, if $\gamma$ as above the critical value.

For the standard Melitz (2003) case ( $\gamma=1$ ), Felbermayr et al. (2013) discuss the role of
the freeness of trade for optimal trade policy. They find that the optimal tariff is increasing in the freeness of trade, such that a fall in non-tariff trade barriers commands a higher optimal tariff. Recall from our previous analysis that from the perspective of the input distortion, $t$ can be decreasing in $\eta$, if the input distortion is dominant. The reason that the import subsidy is less efficient in addressing the input distortion, if the freeness of trade is low, as only a small fraction of varieties is imported. We show in the appendix that with an inactive selection effect $(\theta \rightarrow \sigma-1)$, for $\gamma=1 /(\sigma+1)$ a tariff would be optimal when $\eta<1 .{ }^{29}$ Hence, the optimal tariff can also decrease in the freeness of trade, if $\gamma$ is sufficiently small.

Figure 5 illustrates optimal tariffs as a function of $\gamma$ and $\eta$ for $\sigma=3.8$ and $\theta=\sigma-1$. Note that under this parametrization, the critical $\gamma$ for which laissez faire is optimal is $\gamma=$ $1 /(\sigma+1) \approx 0.2$, which is the lowest $\gamma$ we consider in Figure 4. The maximum tariff emerges from $\gamma=1$ and $\eta=1$. We know from Felbermayr et al. (2003) that the optimal is always lower when $\eta<1$. This can seen from Figure 4 when looking at the profile for $\gamma=1$. Our analysis above has shown that the optimal tariff is also lower in the presence of input-output linkages, potentially turning into an import subsidy. This can be seen in Figure 4 when looking at the profile for $\eta=1$. We have also proven that the $t$ is decreasing in $\eta$ when $\gamma$ is sufficiently small. This can be seen in Figure 4 when looking at the profile for the lowest $\gamma$ in the graph. For intermediate values of $\gamma$, we find an inverted U-shape, i.e., the optimal tariff increases in $\eta$ for small values of $\eta$ and falls in $\eta$ for large values of $\eta$. This result is an implication of the observation that the optimal tariff falls faster in $\gamma$ than in $\eta$.

## 5 Conclusion

In this paper we have analyzed optimal commercial policies in a version of the Melitz (2003) model with input-output linkages. We find that input-output linkages in combination with monopolistic competition give rise to an input distortion which is not present in models with perfect competition or models that ignore input-output linkages. In the laissez-faire equilibrium, firms use too much labor and too little material input. From this perspective, the model

[^20]Figure 5: The role of the labor cost share and the freeness of trade

calls for more use of intermediate inputs. One straightforward implication would be to allow for vertical integration of suppliers to circumvent the double marginalization problem. Given that in our setting production requires inputs from a continuum of suppliers located around the world, however, this strategy is not feasible.

The optimal cooperative policy is a uniform subsidy on domestically produced and imported varieties that exactly offsets the monopolistic markup producers charge over marginal cost. If one is interesting in calculating of welfare losses from non-cooperative policies (e.g., trade policy), the relevant benchmark is one with efficient levels of subsidies and not free trade, as typically assumed.

We show that the welfare gains of moving from market outcomes to the efficient outcome can be substantial. Welfare gains come from a more efficient allocation of the composite good to final and intermediate use and not from reallocation of resources across firms.

Regarding non-cooperative trade policy, we find that the input distortion counteracts the standard terms-of-trade externality, potentially resulting in an optimal import subsidy. Whether an import subsidy or a tariff is optimal depends on the labor cost share and the "freeness of trade" in non-policy measures. It also will depend on the degree of firm heterogeneity, but we have not explored the role of heterogeneity yet. We find that optimal tariffs may fall in real trade costs, if the labor cost share is sufficiently small. This finding questions the importance of the World Trade Organization, as in world with falling trade costs tariff
would have been reduced anyway.
We conclude by pointing out limitations of the present work. First, in our analysis countries are restricted to be symmetric in their elasticities of substitution. It would be interesting to analyze the case where the elasticities between domestic and foreign varieties differ as in Costinot et al. (2016). Then, a uniform treatment of domestic and imported varieties cannot be optimal. Second, restricting the government to a single instrument (trade policy) in the presence of two distortions cannot lead to efficient outcomes. In future work, we will characterize the optimal policy mix. Preliminary results suggests that the optimal policy mix is a subsidy on domestically produced intermediate goods and a tax-cum-subsidy on imported goods. Third, our analysis of non-cooperative trade policy ignores retaliation. We intend to characterize non-cooperative Nash tariffs and to compute welfare losses in the vein of Felbermayr et al. (2013) and Ossa (2016).

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## A Detailed derivations

## A. 1 Zero profit condition

Taking (16), we insert for profits using (12) and for quantities $\tilde{q}_{j i}(\varphi)$ from (14) which yields

$$
\int_{\varphi_{j i}^{*}} \pi(\varphi) g(\varphi) \mathrm{d} \varphi=\frac{\sigma^{-\sigma}}{(\sigma-1)^{1-\sigma}} \frac{\left(\tau_{j i} x_{j}\right)^{1-\sigma}}{\left(1+t_{j i}\right)^{\sigma}} Y_{i} \tilde{P}_{i}^{\sigma-1} \int_{\varphi_{j i}^{*}} \varphi^{\sigma-1} g(\varphi) \mathrm{d} \varphi-w_{j} f_{j i} \int_{\varphi_{j i}^{*}} g(\varphi) \mathrm{d} \varphi .
$$

Assuming a Pareto distribution for $\varphi$, we have $G(\varphi)=1-\varphi^{-\theta}$ and $g(\varphi)=\theta \varphi^{-1-\theta}$, whereby we assume $\theta>\sigma-1$. This implies

$$
\begin{equation*}
\int_{\varphi_{j i}^{*}} \varphi^{\sigma-1} g(\varphi) \mathrm{d} \varphi=\theta \int_{\varphi_{j i}^{*}} \varphi^{\sigma-\theta-2} \mathrm{~d} \varphi=\frac{\theta}{\theta-(\sigma-1)}\left(\varphi_{j i}^{*}\right)^{\sigma-\theta-1} . \tag{54}
\end{equation*}
$$

This term, which we simply refer to as $\xi_{j i}$ in the text, represents the selection of firms located in country $j$ into different markets, including the come market. We refer to this and $\int_{\varphi_{j i}^{*}} g(\varphi) \mathrm{d} \varphi=\left(\varphi_{j i}^{*}\right)^{-\theta}$, whence we may write

$$
\int_{\varphi_{j i}^{*}} \pi(\varphi) g(\varphi) \mathrm{d} \varphi=\frac{\sigma^{-\sigma}}{(\sigma-1)^{1-\sigma}} \frac{\left(\tau_{j i} x_{j}\right)^{1-\sigma}}{\left(1+t_{j i}\right)^{\sigma}} Y_{i} \tilde{P}_{i}^{\sigma-1} \frac{\theta}{\theta-(\sigma-1)}\left(\varphi_{j i}^{*}\right)^{\sigma-\theta-1}-w_{j} f_{j i}\left(\varphi_{j i}^{*}\right)^{-\theta} .
$$

Substituting for $\left(\varphi_{j i}^{*}\right)^{\sigma-1}$ according to the first line of (17), we have

$$
\left(\varphi_{j i}^{*}\right)^{\sigma-1}=\frac{\sigma^{\sigma}}{(\sigma-1)^{\sigma-1}}\left(x_{j} \tau_{j i}\right)^{\sigma-1}\left(1+t_{j i}\right)^{\sigma} \frac{Y_{i}}{\tilde{P}_{i}^{\sigma-1}} w_{j} f_{j i},
$$

whence expected profits of selling from $j$ to $i$ reduce to

$$
\int_{\varphi_{j i}^{*}} \pi(\varphi) g(\varphi) \mathrm{d} \varphi=\frac{\theta}{\theta-(\sigma-1)} w_{j} f_{j i}\left(\varphi_{j i}^{*}\right)^{-\theta}-w_{j} f_{j i} \varphi_{j i}^{*-\theta}=\frac{\sigma-1}{\theta-(\sigma-1)} w_{j} f_{j i}\left(\varphi_{j i}^{*}\right)^{-\theta} .
$$

Summing up over all markets (countries) $i=1, \ldots, M$ and inserting into (16) leads to equation (19).

## A. 2 Price index and expenditure shares

Using the markup pricing condition for firm $\varphi$ of country $j$ when selling to country $i$ as given in (11), the price index for goods assembly given in (5) emerges as

$$
\begin{align*}
\tilde{P}_{i} & =\left(\sum_{j=1}^{M} N_{j} \int_{\varphi_{j i}^{* i}}\left(\frac{\sigma}{\sigma-1} \frac{\left(1+t_{j i}\right) \tau_{j i} x_{j}}{\varphi}\right)^{1-\sigma} g(\varphi) \mathrm{d} \varphi\right)^{\frac{1}{1-\sigma}} \\
& =\left(\sum_{j=1}^{M} N_{j}\left(\frac{\sigma}{\sigma-1}\left(1+t_{j i}\right) \tau_{j i} x_{j}\right)^{1-\sigma} \int_{\varphi_{j i}^{*}} \varphi^{\sigma-1} g(\varphi) \mathrm{d} \varphi\right)^{\frac{1}{1-\sigma}} \\
& =\left(\sum_{j=1}^{M} N_{j} \chi_{j i} \xi_{j i}\right)^{\frac{1}{1-\sigma}}, \tag{55}
\end{align*}
$$

where $\chi_{j i}:=\left(\frac{\sigma}{\sigma-1}\left(1+t_{j i}\right) \tau_{j i} x_{j}\right)^{1-\sigma}$ and $\xi_{j i}:=\int_{\varphi_{j i}^{*}} \varphi^{\sigma-1} g(\varphi) \mathrm{d} \varphi$. Using (54), we may write

$$
\tilde{P}_{i}^{1-\sigma}=\frac{\theta}{\theta-(\sigma-1)} \sum_{j=1}^{M} N_{j} \chi_{j i}\left(\varphi_{j i}^{*}\right)^{\sigma-\theta-1}
$$

From (6) it follows that country $i$ 's expenditure on goods originating in country $j$ may be written as

$$
N_{j} \int_{\varphi_{j i}^{*}} \tilde{p}_{j i}(\varphi) \tilde{q}_{j i}(\varphi) g(\varphi) \mathrm{d} \varphi=N_{j} \int_{\varphi_{j i}^{*}} \tilde{p}_{j i}(\varphi)^{1-\sigma} g(\varphi) \mathrm{d} \varphi \times Y_{i} \tilde{P}_{i}^{\sigma-1}
$$

Inserting from (11) and (13), we have

$$
\begin{aligned}
N_{j} \int_{\varphi_{j i}^{*}} \tilde{p}_{j i}(\varphi) \tilde{q}_{j i}(\varphi) g(\varphi) \mathrm{d} \varphi & =N_{j}\left(\frac{\sigma}{\sigma-1} \tau_{j i} x_{j}\left(1+t_{j i}\right)\right)^{1-\sigma} \int_{\varphi_{j i}^{*}} \varphi^{\sigma-1} g(\varphi) \mathrm{d} \varphi \times Y_{i} \tilde{P}_{i}^{\sigma-1} \\
& =N_{j} \chi_{j i} \xi_{j i} \times Y_{i} \tilde{P}_{i}^{\sigma-1}
\end{aligned}
$$

Forming expenditures shares and using (55), we have

$$
\lambda_{j i}:=\frac{N_{j} \int_{\varphi_{j i}^{*}} \tilde{p}_{j i}(\varphi) \tilde{q}_{j i}(\varphi) g(\varphi) d \varphi}{\sum_{n=1}^{I} N_{n} \int_{\varphi_{n i}^{*}} \tilde{p}_{n i}(\varphi) \tilde{q}_{n i}(\varphi) g(\varphi) d \varphi}=\frac{N_{j} \chi_{j i} \xi_{j i}}{\tilde{P}_{i}^{1-\sigma}}
$$

## A. 3 The income multiplier

From equation (2) and $T_{j}=Y_{j} \sum_{i} \frac{t_{i j} \lambda_{i j}}{1+t_{i j}}$, and using (35), we have

$$
I_{j}=\left(1+\tilde{\mu}_{j} \sum_{i} \frac{t_{i j} \lambda_{i j}}{1+t_{i j}}\right) \times w_{j} L_{j}
$$

This can be rewritten as

$$
\begin{aligned}
I_{j} & =\tilde{\mu}_{j}\left[\left(1-(1-\gamma) \frac{\sigma-1}{\sigma}\right) \sum_{i} \frac{\lambda_{i j}}{1+t_{i j}}+\sum_{i} \frac{t_{i j} \lambda_{i j}}{1+t_{i j}}\right] \times w_{i} L_{i} \\
& =\tilde{\mu}_{j} \sum_{i}\left(1+t_{i j}-(1-\gamma) \frac{\sigma-1}{\sigma}\right) \frac{\lambda_{i j}}{1+t_{i j}} \times w_{i} L_{i}
\end{aligned}
$$

## A. 4 Mass of entrants

We now use the equilibrium conditions derived above to solve for the mass of entrants. The zero cutoff profit condition can be written as

$$
\begin{aligned}
\varphi_{i j}^{*} & =\frac{\sigma}{\sigma-1}\left(\frac{\sigma w_{i} f_{i j}\left(1+t_{i j}\right)}{Y_{j}}\right)^{\frac{1}{\sigma-1}} \frac{\tau_{i j} x_{i}\left(1+t_{i j}\right)}{\tilde{P}_{j}} \Leftrightarrow \\
\left(\varphi_{i j}^{*}\right)^{\sigma-1} & =\left(\frac{\sigma}{\sigma-1}\right)^{\sigma-1}\left(\frac{\sigma w_{i} f_{i j}\left(1+t_{i j}\right)}{Y_{j}}\right)\left(\frac{\tau_{i j} x_{i}\left(1+t_{i j}\right)}{\tilde{P}_{j}}\right)^{\sigma-1} \Leftrightarrow \\
\left(\frac{\sigma}{\sigma-1} \frac{\tau_{i j} x_{i}\left(1+t_{i j}\right)}{\tilde{P}_{j} \varphi_{i j}^{*}}\right)^{1-\sigma} & =\frac{\sigma w_{i} f_{i j}\left(1+t_{i j}\right)}{Y_{j}}
\end{aligned}
$$

Using this expression in the expression for the expenditure share (22), we have

$$
\begin{aligned}
\lambda_{i j} & =\frac{\theta}{\theta-(\sigma-1)} N_{i}\left(\varphi_{i j}^{*}\right)^{-\theta}\left(\frac{\sigma}{\sigma-1} \frac{\left(1+t_{i j}\right) \tau_{i j} x_{i}}{\varphi_{i j}^{*}} \frac{1}{\tilde{P}_{j}}\right)^{1-\sigma} \\
& =\frac{\theta}{\theta-(\sigma-1)} N_{i}\left(\varphi_{i j}^{*}\right)^{-\theta} \frac{\sigma w_{i} f_{i j}\left(1+t_{i j}\right)}{Y_{j}} \Leftrightarrow \\
\frac{\lambda_{i j}}{1+t_{i j}} Y_{j} & =\frac{\theta \sigma}{\theta-(\sigma-1)} N_{i}\left(\varphi_{i j}^{*}\right)^{-\theta} w_{i} f_{i j}
\end{aligned}
$$

Summing both sides over $j$ and solving for $N_{i}$, we have

$$
\begin{aligned}
\sum_{j} \frac{\lambda_{i j}}{1+t_{i j}} Y_{j} & =\frac{\theta \sigma}{\theta-(\sigma-1)} N_{i} w_{i} \sum_{j}\left(\varphi_{i j}^{*}\right)^{-\theta} f_{i j} \\
& =\frac{\theta \sigma}{\theta-(\sigma-1)} N_{i} w_{i} \frac{\theta-(\sigma-1)}{\sigma-1} f_{i}^{e} \Leftrightarrow \\
N_{i} & =\frac{\sigma-1}{\theta \sigma w_{i} f_{i}^{e}} \sum_{j} \frac{\lambda_{i j}}{1+t_{i j}} Y_{j}
\end{aligned}
$$

where the second line follows from (19).
Using the trade balance condition (31) and (32), we obtain

$$
N_{i}=\frac{\sigma-1}{\theta \sigma w_{i} f_{i}^{e}} Y_{i} \sum_{j} \frac{\lambda_{j i}}{1+t_{j i}}=\frac{\sigma-1}{\theta \sigma f_{i}^{e}} \frac{Z_{i}}{w_{i}} .
$$

The expression in the text follows from using (33).

## A. 5 Domestic expenditure share

In order to derive the expression for the domestic expenditure share, we compute $\tilde{P}_{i}^{1-\sigma}$ :

$$
\begin{aligned}
& \tilde{P}_{i}^{1-\sigma}=\frac{\theta}{\theta-(\sigma-1)} \sum_{j=1}^{M} N_{j}\left(\varphi_{j i}^{*}\right)^{-\theta}\left(\frac{\sigma}{\sigma-1} \frac{\left(1+t_{j i}\right) \tau_{j i} x_{j}}{\varphi_{j i}^{*}}\right)^{1-\sigma} \\
& =\frac{\theta\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}}{\theta-(\sigma-1)^{1}} N_{i}\left(\varphi_{i i}^{*}\right)^{\sigma-1-\theta}\left(1+t_{i i}\right)^{1-\sigma} x_{i}^{1-\sigma}\left(1+\sum_{j=1}^{M} N_{j} \frac{N_{j}}{N_{i}}\left(\frac{\varphi_{j i}^{*}}{\varphi_{i i}^{i}}\right)^{\sigma-1-\theta} \tau_{j i j}^{1-\sigma}\left(\frac{1+t_{j i}}{1+t_{i i}}\right)^{1-\sigma}\left(\frac{x_{j}}{x_{i}}\right)^{1-\sigma}\right)
\end{aligned}
$$

$$
\begin{aligned}
& =\frac{\theta\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}}{\theta-(\sigma-1)^{N_{i}}\left(\varphi_{i i}^{\left(\alpha_{i}\right)}\right)^{\sigma-1-\theta}}\left(1+t_{i i}\right)^{1-\sigma} x_{i}^{1-\sigma}\left(1+\sum_{j=1}^{M} \frac{N_{j}}{N_{i}}\left(\left(\frac{w_{j}}{w_{i}}\right)^{\frac{1}{\sigma-1}}\left(\frac{f_{j i}}{f_{i i}}\right)^{\frac{1}{\sigma-1}}\left(\frac{1+t_{j i}}{1+t_{i i}}\right)^{\frac{1}{\sigma-1}+1} \tau_{j j i}^{x_{j}}\right)^{\sigma-1-\theta}{ }_{\tau j i}^{\frac{1}{1-\sigma}}\left(\frac{1+t_{j i}}{1+t_{i i}}\right)^{1-\sigma}\left(\frac{x_{j}}{x_{i}}\right)^{1-\sigma}\right) \\
& =\frac{\theta\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}}{\theta-(\sigma-1)^{N}\left(\varphi_{i j}^{*}\right)^{\sigma-1-\theta}\left(1+t_{i i}\right)^{1-\sigma} x_{i}^{1-\sigma}}\left(1+\sum_{j=1}^{M} \frac{N_{j}}{N_{i}} \tau_{j i}^{-\theta}\left(\frac{1+t_{j i}}{1+t_{i i}}\right)^{1-\sigma+\frac{\theta(\sigma-1-\theta)}{\sigma-1}}\left(\frac{f_{i j}}{f_{i i}}\right)^{\frac{\sigma-1-\theta}{\sigma-1}}\left(\frac{w_{j}}{w_{i}}\right)^{\frac{\sigma-1-\theta}{\sigma-1}}\left(\frac{x_{j}}{x_{i}}\right)^{-\theta}\right) \\
& =\frac{\theta\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}}{\theta-(\sigma-1)^{N}} N_{i}\left(\varphi_{i i}^{*}\right)^{\sigma-1-\theta}\left(1+t_{i i^{2}}^{1-\sigma} x_{i}^{1-\sigma}\left(1+\sum_{j=1}^{M} \frac{N_{j}}{N_{i}-\tau_{i j}}\left(\frac{1+t_{i j}}{1+t_{i i}}\right)^{1-\frac{\sigma \theta}{\sigma-1}}\left(\frac{f_{j i}}{f_{i i}}\right)^{\frac{\sigma-1-\theta}{\sigma-1}}\left(\frac{w_{j}}{w_{i}}\right)^{\frac{\sigma-1-\theta}{\sigma-1}}\left(\frac{x_{j}}{x_{i}}\right)^{-\theta}\right)\right.
\end{aligned}
$$

Hence, the ratio of input cost to the price index is implied by

$$
\begin{aligned}
\left(\frac{x_{i}}{\tilde{P}_{i}}\right)^{\theta}= & \frac{\theta\left(\frac{\sigma}{\sigma-1}\right)^{-\theta}\left(\sigma f_{i i}\right)^{\frac{\sigma-1-\theta}{\sigma-1}}}{\theta-(\sigma-1)} N_{i}\left(\frac{w_{i}}{Y_{i}}\right)^{\frac{\sigma-1-\theta}{\sigma-1}}\left(1+t_{i i}\right)^{1-\frac{\sigma \theta}{\sigma-1}} \\
& \times\left(1+\sum_{j=1}^{M} \frac{N_{j}}{N_{i}} \tau_{j i}^{-\theta}\left(\frac{1+t_{j i}}{1+t_{i i}}\right)^{1-\frac{\sigma \theta}{\sigma-1}}\left(\frac{f_{j i}}{f_{i i}}\right)^{\frac{\sigma-1-\theta}{\sigma-1}}\left(\frac{w_{j}}{w_{i}}\right)^{\frac{\sigma-1-\theta}{\sigma-1}}\left(\frac{x_{j}}{x_{i}}\right)^{-\theta}\right) .
\end{aligned}
$$

Using this expression to substitute out the ratio from the domestic expenditure share, we obtain

$$
\lambda_{i i}=\left(1+\sum_{j=1}^{M} \frac{N_{j}}{N_{i}} \tau_{j i}^{-\theta}\left(\frac{1+t_{j i}}{1+t_{i i}}\right)^{1-\frac{\sigma \theta}{\sigma-1}}\left(\frac{f_{j i}}{f_{i i}}\right)^{\frac{\sigma-1-\theta}{\sigma-1}}\left(\frac{w_{j}}{w_{i}}\right)^{\frac{\sigma-1-\theta}{\sigma-1}}\left(\frac{x_{j}}{x_{i}}\right)^{-\theta}\right)^{-1}
$$

Substituting out the input cost index, we obtain

$$
\begin{aligned}
\left(\frac{x_{j}}{x_{i}}\right)^{-\theta} & =\left(\frac{w_{j}}{w_{i}}\right)^{\frac{\sigma-1-\theta}{\sigma-1}}\left(\frac{w_{j}^{\gamma} \tilde{P}_{j}^{1-\gamma}}{w_{i}^{\gamma} \tilde{P}_{i}^{1-\gamma}}\right)^{-\theta} \\
& =\left(\frac{w_{j}}{w_{i}}\right)^{-\theta}\left(\frac{w_{j}^{\gamma-1} \tilde{P}_{j}^{1-\gamma}}{w_{i}^{\gamma-1} \tilde{P}_{i}^{1-\gamma}}\right)^{-\theta} \\
& =\left(\frac{w_{j}}{w_{i}}\right)^{-\theta}\left(\frac{\xi_{j}}{\xi_{i}}\left(\frac{N_{j}}{N_{i}}\right)^{\frac{1}{\gamma \theta}}\left(\frac{\tilde{\mu}_{j}}{\tilde{\mu}_{i}}\right)^{\frac{\theta-.(\sigma-1)}{\gamma \theta(\sigma-1)}}\left(\frac{1+t_{j j}}{1+t_{i i}}\right)^{-\frac{1}{\gamma \theta}\left(\frac{\sigma \theta}{\sigma-1}-1\right)}\left(\frac{\lambda_{j j}}{\lambda_{i i}}\right)^{-\frac{1}{\gamma \theta}}\right)^{(1-\gamma) \theta} \\
& =\left(\frac{w_{j}}{w_{i}}\right)^{-\theta}\left(\left(\frac{\xi_{j}}{\xi_{i}}\right)^{\theta \gamma} \frac{N_{j}}{N_{i}}\left(\frac{\tilde{\mu}_{j}}{\tilde{\mu}_{i}}\right)^{\frac{\theta-.(\sigma-1)}{\sigma-1}}\left(\frac{1+t_{j j}}{1+t_{i i}}\right)^{1-\frac{\sigma \theta}{\sigma-1}}\left(\frac{\lambda_{j j}}{\lambda_{i i}}\right)^{-1}\right)^{\frac{1-\gamma}{\gamma}}
\end{aligned}
$$

## A. 6 Selection with uniform treatment of domestically and imported goods

We start from the zero cutoff profit condition

$$
\varphi_{j i}^{*}=\frac{\sigma^{\frac{\sigma}{\sigma-1}}}{\sigma-1} f_{j i}^{\frac{1}{\sigma-1}} \tau_{j i}\left(1+t_{j i}\right)^{\frac{\sigma}{\sigma-1}} x_{j} w_{j}^{\frac{1}{\sigma-1}} \frac{Y_{i}^{\frac{1}{1-\sigma}}}{\tilde{P}_{i}} .
$$

Substituting out gross output, we obtain

$$
\varphi_{j i}^{*}=\frac{\sigma^{\frac{\sigma}{\sigma-1}}\left(L_{i}\right)^{\frac{1}{1-\sigma}}}{\sigma-1} f_{j i}^{\frac{1}{\sigma-1}} \tau_{j i}\left(1+t_{j i}\right)_{j}^{\frac{\sigma}{\sigma-1}} x_{j} \frac{\tilde{\mu}_{i}^{\frac{1}{1-\sigma}}}{\tilde{P}_{i}}\left(\frac{w_{j}}{w_{i}}\right)^{\frac{1}{\sigma-1}} .
$$

Next, we employ the definition of the input cost index $x_{j}$ :

$$
\varphi_{j i}^{*}=A \frac{\sigma^{\frac{\sigma}{\sigma-1}} L_{i}^{\frac{1}{1-\sigma}}}{\sigma-1} f_{j i}^{\frac{1}{\sigma-1}} \tau_{j i}\left(1+t_{j i}\right)_{j}^{\frac{\sigma}{\sigma-1}} \tilde{\mu}_{i}^{\frac{1}{1-\sigma}}\left(\frac{w_{j}}{\tilde{P}_{j}}\right)^{\gamma} \frac{\tilde{P}_{j}}{\tilde{P}_{i}}\left(\frac{w_{j}}{w_{i}}\right)^{\frac{1}{\sigma-1}} .
$$

Using equation (40) to substitute out the real wage, we obtain

$$
\varphi_{j i}^{*}=\zeta j \frac{\sigma^{\frac{\sigma}{\sigma-1}} L_{i}^{\frac{1}{1-\sigma}}}{\sigma-1} N_{j}^{\frac{1}{\theta}} f_{j i}^{\frac{1}{\sigma-1}} \tau_{j i}\left(\frac{w_{j}}{w_{i}}\right)^{\frac{1}{\sigma-1}} \frac{\tilde{P}_{j}}{\tilde{P}_{i}}\left(1+t_{j i}\right)^{\frac{\sigma}{\sigma-1}} \tilde{\mu}_{i}^{\frac{1}{1-\sigma}}\left(\tilde{\mu}_{j}^{\frac{\theta-(\sigma-1)}{\sigma-1}}\left(1+t_{j j}\right)^{1-\frac{\sigma \theta}{\sigma-1}}\right)^{\frac{1}{\theta}} \lambda_{j j}^{-\frac{1}{\theta}} .
$$

With symmetric treatment of domestically produced and imported goods, we have

$$
\begin{aligned}
\varphi_{j i}^{*} & \propto \zeta j \frac{\sigma^{\frac{\sigma}{\sigma-1}} L_{i}^{\frac{1}{1-\sigma}}}{\sigma-1} N_{j}^{\frac{1}{\theta}} f_{j i}^{\frac{1}{\sigma-1}} \tau_{j i}\left(\frac{w_{j}}{w_{i}}\right)^{\frac{1}{\sigma-1}} \frac{\tilde{P}_{j}}{\tilde{P}_{i}}\left(1+t_{j i}\right)^{\frac{\sigma}{\sigma-1}} \tilde{\mu}_{i}^{\frac{1}{1-\sigma}}\left(1+t_{j}\right)^{-1} \lambda_{j j}^{-\frac{1}{\theta}} \\
& \propto \zeta j \frac{\sigma^{\frac{\sigma}{\sigma-1}} L_{i}^{\frac{1}{1-\sigma}}}{\sigma-1} N_{j}^{\frac{1}{\theta}} f_{j i}^{\frac{1}{\sigma-1}} \tau_{j i}\left(\frac{w_{j}}{w_{i}}\right)^{\frac{1}{\sigma-1}} \frac{\tilde{P}_{j}}{\tilde{P}_{i}} \frac{1+t_{i}}{1+t_{j}} \lambda_{j j}^{-\frac{1}{\theta}} .
\end{aligned}
$$

This expression highlights that with symmetric countries, cutoffs are invariant to changes in commercial policy.

With asymmetric countries, we need some more steps to that the same claim holds under the additional restriction that all countries conduct the same policy. The condition above can be rewritten as

$$
\varphi_{j i}^{*} \propto \zeta j \frac{\sigma^{\frac{\sigma}{\sigma-1}} L_{i}^{\frac{1}{1-\sigma}}}{\sigma-1} N_{j}^{\frac{1}{\theta}} f_{j i}^{\frac{1}{\sigma-1}} \tau_{j i}\left(\frac{w_{j}}{w_{i}}\right)^{\frac{1}{\sigma-1}-1}\left(\frac{w_{j} / \tilde{P}_{j}}{w_{i} \tilde{P}_{i}}\right) \frac{1+t_{i}}{1+t_{j}} \lambda_{j j}^{-\frac{1}{\theta}} .
$$

Using again equation (40), we obtain

$$
\begin{aligned}
\varphi_{j i}^{*} \propto & \zeta j \frac{\sigma^{\frac{\sigma}{\sigma-1}} L_{i}^{\frac{1}{1-\sigma}}}{\sigma-1} N_{j}^{\frac{1}{\theta}} f_{j i}^{\frac{1}{\sigma-1}} \tau_{j i}\left(\frac{w_{j}}{w_{i}}\right)^{\frac{2-\sigma}{\sigma-1}} \frac{\zeta_{j}}{\zeta_{i}}\left(\frac{N_{j}}{N_{i}}\right)^{\frac{1}{\gamma \theta}}\left(\frac{\tilde{\mu}_{j}}{\tilde{\mu}_{i}}\right)^{\frac{\theta-(\sigma-1)}{\gamma \theta(\sigma-1)}}\left(\frac{1+t_{j}}{1+t_{i}}\right)^{\frac{1}{\gamma \theta}\left(1-\frac{\sigma \theta}{\sigma-1}\right)} \\
& \times\left(\frac{\lambda_{j j}}{\lambda_{i i}}\right)^{-\frac{1}{\gamma \theta}} \frac{1+t_{i}}{1+t_{j}} \lambda_{j j}^{-\frac{1}{\theta}} \\
\propto & \zeta j \frac{\frac{\sigma}{\sigma-1}}{\sigma-1} L_{i}^{\frac{1}{1-\sigma}} N_{j}^{\frac{1}{\theta}} f_{j i}^{\frac{1}{\sigma-1}} \tau_{j i}\left(\frac{w_{j}}{w_{i}}\right)^{\frac{2-\sigma}{\sigma-1}} \frac{\zeta_{j}}{\zeta_{i}}\left(\frac{N_{j}}{N_{i}}\right)^{\frac{1}{\gamma \theta}}\left(\frac{1+t_{j}}{1+t_{i}}\right)^{-\frac{1}{\gamma}}\left(\frac{\lambda_{j j}}{\lambda_{i i}}\right)^{-\frac{1}{\gamma \theta}} \frac{1+t_{i}}{1+t_{j}} \lambda_{j j}^{-\frac{1}{\theta}} .
\end{aligned}
$$

Recognizing that with uniform treatment domestic expenditure shares and relative wages are determined independently of commercial policy, cutoff productivity levels are also deter-
mined independently of commercial policies in the presence of country asymmetries, under the precondition that all countries pursue the same commercial policy.

## A. 7 Efficiency gains of moving from laissez-fare to social optimum

Taking the derivative of the welfare differential with respect to $\rho:=\frac{\sigma-1}{\sigma}$, we obtain

$$
\begin{aligned}
\frac{\partial \frac{W^{*}}{W^{f}}}{\partial \rho} & =\gamma \frac{-\frac{1-\gamma}{\gamma} \rho^{-\frac{1-\gamma}{\gamma}-1}[1-(1-\gamma) \rho]-\rho^{-\frac{1-\gamma}{\gamma}}(1-\gamma)}{[1-(1-\gamma)]^{2}} \\
& =-(1-\gamma) \rho^{-\frac{1-\gamma}{\gamma}-1} \frac{1-(1-\gamma) \rho+\rho \gamma}{[1-(1-\gamma)]^{2}} \\
& =-\frac{2 \gamma(1-\gamma) \rho^{-\frac{1-\gamma}{\gamma}}}{[1-(1-\gamma)]^{2}}<0 .
\end{aligned}
$$

Taking the derivative with respect to $\gamma$, we obtain

$$
\begin{aligned}
\frac{\partial \frac{W^{*}}{W^{l}}}{\partial \gamma} & =\frac{\left(\rho^{-\frac{1-\gamma}{\gamma}}-\gamma \ln (\rho) \rho^{-\frac{1-\gamma}{\gamma}} \frac{-\gamma-(1-\gamma)}{\gamma^{2}}\right)[1-(1-\gamma) \rho]-\gamma \rho^{-\frac{1-\gamma}{\gamma}} \rho}{[1-(1-\gamma) \rho]^{2}} \\
& =\rho^{-\frac{1-\gamma}{\gamma}} \frac{\left(1+\frac{\ln (\rho)}{\gamma}\right)[1-(1-\gamma) \rho]-\gamma \rho}{[1-(1-\gamma) \rho]^{2}} \\
& =\rho^{-\frac{1-\gamma}{\gamma}} \frac{1-(1-\gamma) \rho+\frac{\ln (\rho)}{\gamma}[1-(1-\gamma) \rho]-\gamma \rho}{[1-(1-\gamma) \rho]^{2}} \\
& =\rho^{-\frac{1-\gamma}{\gamma}} \frac{1-\rho+\gamma \rho+\frac{\ln (\rho)}{\gamma}[1-(1-\gamma) \rho]-\gamma \rho}{[1-(1-\gamma) \rho]^{2}} \\
& =\rho^{-\frac{1-\gamma}{\gamma}} \frac{1-\rho+\frac{\ln (\rho)}{\gamma}[1-(1-\gamma) \rho]}{[1-(1-\gamma) \rho]^{2}} .
\end{aligned}
$$

Evaluated at $\gamma=1$, we have

$$
\left.\frac{\partial \frac{W^{*}}{W^{\text {f }}}}{\partial \gamma}\right|_{\gamma=1}<0 \Leftrightarrow 1+\ln (\rho)<\rho
$$

which always holds. In general, we have

$$
\frac{\partial \frac{W^{*}}{W^{\prime f}}}{\partial \gamma}<0 \Leftrightarrow 1+\ln (\rho) \frac{1-(1-\gamma) \rho}{\gamma}-\rho<0 .
$$

We numerically check whether the inequality holds for all combinations of $\gamma \in(0,1)$ and $\rho \in(0,1)$. We find that the inequality always holds. Hence, the welfare gains of moving from laissez-faire to social optimum is larger, the smaller $\gamma$.

## A. 8 Optimal cooperative trade policy

The gross output multiplier is given by

$$
\begin{aligned}
\tilde{\mu} & \equiv\left(1-(1-\gamma) \frac{\sigma-1}{\sigma}\right)^{-1}\left(\lambda+\frac{1-\lambda}{1+t}\right)^{-1} \\
& =\kappa \frac{1+t}{(1+t) \lambda+1-\lambda}=\kappa \frac{1+t}{1+t \lambda}=\kappa\left(1+\frac{1-\lambda}{1+\lambda t} t\right)
\end{aligned}
$$

Totally differentiating this expression, we obtain

$$
\begin{aligned}
\frac{d \tilde{\mu}}{\tilde{\mu}} & =\frac{d t}{1+t}-\frac{\lambda}{1+t \lambda} d t-\frac{t \lambda}{1+t \lambda} \frac{d \lambda}{\lambda} \\
& =\frac{\lambda t+1-(1+t) \lambda}{(1+t)(1+t \lambda)} d t-\frac{t \lambda}{1+t \lambda} \frac{d \lambda}{\lambda} \\
& =\frac{1-\lambda}{(1+t)(1+t \lambda)} d t-\frac{t \lambda}{1+t \lambda} \frac{d \lambda}{\lambda}
\end{aligned}
$$

Evaluated at free trade $(t=0)$, we have

$$
\left.\frac{d \tilde{\mu} / \tilde{\mu}}{d t}\right|_{\vartheta=t}=1-\lambda>0 .
$$

The income multiplier is given by

$$
\begin{aligned}
\mu & \equiv \tilde{\mu}\left[\left(1-(1-\gamma) \frac{\sigma-1}{\sigma}\right) \lambda+\left(1+t-(1-\gamma) \frac{\sigma-1}{\sigma}\right) \frac{1-\lambda}{1+t}\right] \\
& =\frac{\tilde{\mu}}{1+t}\left[\left(1-(1-\gamma) \frac{\sigma-1}{\sigma}\right) \lambda(1+t)+\left(1+t-(1-\gamma) \frac{\sigma-1}{\sigma}\right)(1-\lambda)\right] \\
& =\frac{\tilde{\mu}}{1+t}\left[\lambda(1+t)-(1-\gamma) \frac{\sigma-1}{\sigma} \lambda(1+t)+(1+t)(1-\lambda)-(1-\gamma) \frac{\sigma-1}{\sigma}(1-\lambda)\right] \\
& =\frac{\tilde{\mu}}{1+t}\left[1+t-(1-\gamma) \frac{\sigma-1}{\sigma} \lambda(1+t)-(1-\gamma) \frac{\sigma-1}{\sigma}(1-\lambda)\right] \\
& =\frac{\tilde{\mu}}{1+t}\left[1+t-(1-\gamma) \frac{\sigma-1}{\sigma} \lambda-(1-\gamma) \frac{\sigma-1}{\sigma} \lambda t-(1-\gamma) \frac{\sigma-1}{\sigma}+(1-\gamma) \frac{\sigma-1}{\sigma} \lambda\right] \\
& =\tilde{\mu} \frac{\lambda t+1}{1+t}\left[\frac{1+t}{1+\lambda t}-(1-\gamma) \frac{\sigma-1}{\sigma}\right] \\
& =\kappa\left[\frac{1+t}{1+t \lambda}-1+1-(1-\gamma) \frac{\sigma-1}{\sigma}\right] \\
& =1+\kappa t \frac{1-\lambda}{1+t \lambda}
\end{aligned}
$$

Totally differentiating this expression, we obtain

$$
\begin{aligned}
\frac{d \mu}{\mu} & =\frac{\frac{t \lambda+1-(1+t) \lambda}{(1+t \lambda)^{2}} d t-\frac{t(1+t)}{(t \lambda+1)^{2}} d \lambda}{\frac{1+t}{1+t \lambda}-(1-\gamma) \frac{\sigma-1}{\sigma}} \\
& =\frac{1}{(1+t \lambda)^{2}} \frac{(1-\lambda) d t-t(1+t) d \lambda}{\frac{1+t}{1+t \lambda}-(1-\gamma) \frac{\sigma-1}{\sigma}} .
\end{aligned}
$$

Evaluated at free trade ( $t=0$ ), we have

$$
\left.\frac{d \mu / \mu}{d t}\right|_{t=0}=\kappa(1-\lambda)
$$

The domestic expenditure share is given by

$$
\lambda=\frac{1}{1+\eta(1+t)^{1-\frac{\sigma \theta}{\sigma-1}}} .
$$

Totally differentiating, we obtain

$$
\frac{d \lambda}{\lambda}=-(1-\lambda)\left(1-\frac{\sigma \theta}{\sigma-1}\right) \frac{d t}{1+t} .
$$

Notice that changes in $t$ affect the domestic expenditure share through an effect on the price (intensive margin) and selection (extensive margin). Evaluated at free trade ( $t=0$ ), we obtain

$$
\left.\frac{d \lambda / \lambda}{d t}\right|_{t=0}=(1-\lambda)\left(\frac{\sigma \theta}{\sigma-1}-1\right) .
$$

In totally differentiated from, welfare is given by

$$
\frac{d W}{W}=\frac{d \mu}{\mu}+\frac{\theta-(\sigma-1)}{\gamma \theta(\sigma-1)} \frac{d \tilde{\mu}}{\tilde{\mu}}-\frac{1}{\gamma \theta} \frac{d \lambda}{\lambda} .
$$

Evaluated at free trade $(t=0)$, a subsidy on imported varieties increases welfare as

$$
\begin{aligned}
\left.\frac{d W / W}{d t}\right|_{t=0} & =\kappa(1-\lambda)+\frac{\theta-(\sigma-1)}{\gamma \theta(\sigma-1)}(1-\lambda)-\frac{1}{\gamma \theta}(1-\lambda)\left(\frac{\sigma \theta}{\sigma-1}-1\right) \\
& =(1-\lambda)\left[\kappa+\frac{1}{\gamma \theta}\left(\frac{\theta-(\sigma-1)}{\sigma-1}-\frac{\sigma \theta}{\sigma-1}+1\right)\right] \\
& =(1-\lambda)\left(\kappa-\frac{1}{\gamma}\right)<0
\end{aligned}
$$

since

$$
\begin{aligned}
\operatorname{sign}\left(\kappa-\frac{1}{\gamma}\right) & =\operatorname{sign}\left(\gamma-1+(1-\gamma) \frac{\sigma-1}{\sigma}\right) \\
& =\operatorname{sign}\left(\frac{\sigma-1}{\sigma}-1\right)=\operatorname{sign}(-1)
\end{aligned}
$$

Hence, an import subsidy is welfare enhancing.
In general, the change in welfare induced by trade policy is given by

$$
\begin{aligned}
& \frac{d W / W}{d t}=\frac{1}{(1+t \lambda)^{2}} \frac{1-\lambda-(1+t) t d \lambda / d t}{\frac{1+t}{1+t \lambda}-(1-\gamma) \frac{\sigma-1}{\sigma}}+\frac{\theta-(\sigma-1)}{\gamma \theta(\sigma-1)}\left(\frac{1-\lambda}{(1+t)(1+t \lambda)}-\frac{t \lambda}{1+t \lambda} \frac{d \lambda / d \vartheta}{\lambda}\right)-\frac{1}{\gamma \theta} \frac{d \lambda / d t}{\lambda} \\
& =\left(\frac{1}{1+t \lambda} \frac{1+t}{\frac{1+t}{1+t \lambda}-(1-\gamma) \frac{\sigma-1}{\sigma}}+\frac{\theta-(\sigma-1)}{\gamma \theta(\sigma-1)}\right) \frac{1-\lambda}{(1+t \lambda)(1+t)}-\left(\frac{1}{(1+t \lambda)^{2}} \frac{(1+t) t \lambda}{\frac{1+\vartheta}{1+t \lambda}-(1-\gamma) \frac{\sigma-1}{\sigma}}+\frac{\theta-(\sigma-1)}{\gamma \theta(\sigma-1)} \frac{t \lambda}{1+t \lambda}+\frac{1}{\gamma \theta}\right) \frac{d \lambda / d t}{\lambda} \\
& =\left[\frac{1}{1+t \lambda} \frac{1+t}{\frac{1+t}{1+t \lambda}-(1-\gamma) \frac{\sigma-1}{\sigma}}+\frac{\theta-(\sigma-1)}{\gamma \theta(\sigma-1)}-\left(\frac{1}{1+t \lambda} \frac{(1+t) t \lambda}{\frac{1+t}{1+t \lambda}-(1-\gamma) \frac{\sigma-1}{\sigma}}+\frac{\theta-(\sigma-1)}{\gamma \theta(\sigma-1)} t \lambda+\frac{1+t \lambda}{\gamma \theta}\right)\left(\frac{\sigma \theta}{\sigma-1}-1\right)\right] \frac{1-\lambda}{(1+t)(1+t \lambda)} \\
& =\left[\frac{1}{1+t \lambda} \frac{1+t}{\frac{1+t}{1+t \lambda}-(1-\gamma) \frac{\sigma-1}{\sigma}}+\frac{\theta-(\sigma-1)}{\gamma \theta(\sigma-1)}-\left(\frac{1}{1+t \lambda} \frac{(1+t) t \lambda}{\frac{1+t}{1+t \lambda}-(1-\gamma) \frac{\sigma-1}{\sigma}}+\frac{1}{\gamma \theta}\left(\frac{\theta t \lambda}{\sigma-1}-t \lambda+t \lambda+1\right)\right)\left(\frac{\sigma \theta}{\sigma-1}-1\right)\right] \frac{1-\lambda}{(1+t)(1+t \lambda)} \\
& =\left[\frac{1}{1+t \lambda} \frac{1+t}{\frac{1+t}{1+t \lambda}-(1-\gamma) \frac{\sigma-1}{\sigma}}+\frac{\theta-(\sigma-1)}{\gamma \theta(\sigma-1)}-\frac{\frac{\sigma \theta}{\sigma-1}-1}{1+t \lambda} \frac{(1+t) t \lambda}{\frac{1+t}{1+t \lambda}-(1-\gamma) \frac{\sigma-1}{\sigma}}-\frac{\frac{\sigma \theta}{\sigma-1}-1}{\gamma \theta}\left(\frac{\theta \vartheta \lambda}{\sigma-1}+1\right)\right] \frac{1-\lambda}{(1+t)(1+t \lambda)} \\
& =\left[\frac{1}{1+t \lambda} \frac{1+t-\left(\frac{\sigma \theta}{\sigma-1}-1\right)(1+t) t \lambda}{\frac{1+t}{1+t \lambda}-(1-\gamma) \frac{\sigma-1}{\sigma}}+\frac{1}{\gamma \theta}\left(\frac{\theta}{\sigma-1}-1-\left(\frac{\sigma \theta}{\sigma-1}-1\right)\left(\frac{\theta t \lambda}{\sigma-1}+1\right)\right)\right] \frac{1-\lambda}{(1+t)(1+t \lambda)} \\
& =\left[\frac{1+t}{1+t \lambda} \frac{1-\left(\frac{\sigma \theta}{\sigma-1}-1\right) t \lambda}{\frac{1+t}{1+t \lambda}-(1-\gamma) \frac{\sigma-1}{\sigma}}+\frac{1}{\gamma \theta}\left(\frac{\theta}{\sigma-1}-1-\left(\frac{\sigma \theta}{\sigma-1}-1\right) \frac{\theta t \lambda}{\sigma-1}-\frac{\sigma \theta}{\sigma-1}+1\right)\right] \frac{1-\lambda}{(1+t)(1+t \lambda)} \\
& =\left[\frac{1+t}{1+t \lambda} \frac{1-\left(\frac{\sigma \theta}{\sigma-1}-1\right) t \lambda}{\frac{1+t}{1+t \lambda}-(1-\gamma) \frac{\sigma-1}{\sigma}}+\frac{1}{\gamma \theta}\left(\frac{\theta}{\sigma-1}-\frac{\sigma \theta}{\sigma-1}-\left(\frac{\sigma \theta}{\sigma-1}-1\right) \frac{\theta t \lambda}{\sigma-1}\right)\right] \frac{1-\lambda}{(1+t)(1+t \lambda)} \\
& =\left[\frac{1+t}{1+t \lambda} \frac{1-\left(\frac{\sigma \theta}{\sigma-1}-1\right) t \lambda}{\frac{1+t}{1+t \lambda}-(1-\gamma) \frac{\sigma-1}{\sigma}}-\frac{1}{\gamma}\left(1+\left(\frac{\sigma \theta}{\sigma-1}-1\right) \frac{t \lambda}{\sigma-1}\right)\right] \frac{1-\lambda}{(1+t)(1+t \lambda)} \\
& =\left[\frac{1+t}{1+t \lambda} \frac{1-\left(\frac{\sigma \theta}{\sigma-1}-1\right) t \lambda}{\frac{1+t}{1+t \lambda}-(1-\gamma) \frac{\sigma-1}{\sigma}}-\frac{1}{\gamma}\left(1+\left(\frac{\sigma \theta}{\sigma-1}-1\right) \frac{t \lambda}{\sigma-1}\right)\right] \frac{1-\lambda}{(1+t)(1+t \lambda)} \\
& =\left[\frac{(1+t)\left(1-\left(\frac{\sigma \theta}{\sigma-1}-1\right) t \lambda\right)}{1+t-(1+t \lambda)(1-\gamma) \frac{\sigma-1}{\sigma}}-\frac{1}{\gamma}\left(1+\left(\frac{\sigma \theta}{\sigma-1}-1\right) \frac{t \lambda}{\sigma-1}\right)\right] \frac{1-\lambda}{(1+t)(1+t \lambda)}
\end{aligned}
$$

Evaluating the welfare change at $t=-1 / \sigma$, we obtain

$$
\begin{aligned}
\left.\frac{d W / W}{d t}\right|_{t=-\frac{1}{\sigma}} & \propto \frac{\left(1-\frac{1}{\sigma}\right)\left(1+\left(\frac{\sigma \theta}{\sigma-1}-1\right) \frac{\lambda}{\sigma}\right)}{1-\frac{1}{\sigma}-\left(1-\frac{\lambda}{\sigma}\right)(1-\gamma) \frac{\sigma-1}{\sigma}}-\frac{1}{\gamma}\left(1-\left(\frac{\sigma \theta}{\sigma-1}-1\right) \frac{\lambda}{\sigma(\sigma-1)}\right) \\
& =\frac{1+\left(\frac{\sigma \theta}{\sigma-1}-1\right) \frac{\lambda}{\sigma}}{1-\left(1-\frac{\lambda}{\sigma}\right)(1-\gamma)}-\frac{1}{\gamma}\left(1-\left(\frac{\sigma \theta}{\sigma-1}-1\right) \frac{\lambda}{\sigma(\sigma-1)}\right) \\
& =\frac{1+\left(\frac{\sigma \theta}{\sigma-1}-1\right) \frac{\lambda}{\sigma}}{1-\left(1-\frac{\lambda}{\sigma}\right)(1-\gamma)}+\frac{1}{\gamma}\left(\frac{\sigma \theta}{\sigma-1}-1\right) \frac{\lambda}{\sigma(\sigma-1)}-\frac{1}{\gamma}
\end{aligned}
$$

Note that the sign of this expression is ambiguous. While first and second terms are certainly positive, the third is negative. If $\gamma$ is sufficiently large, starting from $t=-1 / \sigma$, an increase in $t$, i.e., lowering the rate of the import subsidy, raises welfare. If, to the contrary, $\gamma$ is low, starting from $t=-1 / \sigma$, a further reduction in the import subsidy is welfare enhancing.

In order to further characterize the role of $\gamma$, we rearrange terms:

$$
\begin{aligned}
\left.\frac{d W / W}{d t}\right|_{t=-\frac{1}{\sigma}} & \propto \frac{\gamma+\gamma\left(\frac{\sigma \theta}{\sigma-1}-1\right) \frac{\lambda}{\sigma}}{1-\left(1-\frac{\lambda}{\sigma}\right)(1-\gamma)}-1+\left(\frac{\sigma \theta}{\sigma-1}-1\right) \frac{\lambda}{\sigma(\sigma-1)} \\
& =\frac{\gamma+\gamma\left(\frac{\sigma \theta}{\sigma-1}-1\right) \frac{\lambda}{\sigma}-1+\left(1-\frac{\lambda}{\sigma}\right)(1-\gamma)}{1-\left(1-\frac{\lambda}{\sigma}\right)(1-\gamma)}+\left(\frac{\sigma \theta}{\sigma-1}-1\right) \frac{\lambda}{\sigma(\sigma-1)} \\
& =\frac{\gamma\left(\frac{\sigma \theta}{\sigma-1}-1\right) \frac{\lambda}{\sigma}-(1-\gamma)+\left(1-\frac{\lambda}{\sigma}\right)(1-\gamma)}{1-\left(1-\frac{\lambda}{\sigma}\right)(1-\gamma)}+\left(\frac{\sigma \theta}{\sigma-1}-1\right) \frac{\lambda}{\sigma(\sigma-1)} \\
& =\frac{\gamma\left(\frac{\sigma \theta}{\sigma-1}-1\right) \frac{\lambda}{\sigma}+\left(1-\frac{\lambda}{\sigma}-1\right)(1-\gamma)}{1-\left(1-\frac{\lambda}{\sigma}\right)(1-\gamma)}+\left(\frac{\sigma \theta}{\sigma-1}-1\right) \frac{\lambda}{\sigma(\sigma-1)} \\
& \propto \frac{\gamma\left(\frac{\sigma \theta}{\sigma-1}-1\right)-1+\gamma}{1-\left(1-\frac{\lambda}{\sigma}\right)(1-\gamma)}+\left(\frac{\sigma \theta}{\sigma-1}-1\right) \frac{1}{\sigma-1} \\
& =\frac{\gamma \frac{\sigma \theta}{\sigma-1}-1}{1-\left(1-\frac{\lambda}{\sigma}\right)(1-\gamma)}+\left(\frac{\sigma \theta}{\sigma-1}-1\right) \frac{1}{\sigma-1} \\
& \propto\left(\gamma \frac{\sigma \theta}{\sigma-1}-1\right)(\sigma-1)+\left(\frac{\sigma \theta}{\sigma-1}-1\right)\left[1-\left(1-\frac{\lambda}{\sigma}\right)(1-\gamma)\right] \\
& =\gamma \sigma \theta+1-\sigma+\frac{\sigma \theta}{\sigma-1}-1-\left(\frac{\sigma \theta}{\sigma-1}-1\right)\left(1-\frac{\lambda}{\sigma}\right)(1-\gamma) \\
& =\gamma \sigma \theta-\sigma+\frac{\sigma \theta}{\sigma-1}-\left(\frac{\sigma \theta}{\sigma-1}-1\right)\left(1-\frac{\lambda}{\sigma}\right)(1-\gamma) \\
& =\gamma \sigma \theta-\sigma+\frac{\sigma \theta}{\sigma-1}-\frac{\sigma \theta}{\sigma-1}\left(1-\frac{\lambda}{\sigma}\right)(1-\gamma)+\left(1-\frac{\lambda}{\sigma}\right)(1-\gamma) \\
& =\sigma(\gamma \theta-1)+\frac{\sigma \theta}{\sigma-1}\left[1-\left(1-\frac{\lambda}{\sigma}\right)(1-\gamma)\right]+\left(1-\frac{\lambda}{\sigma}\right)(1-\gamma)
\end{aligned}
$$

Considering the limiting case $\theta \rightarrow \sigma-1$, we obtain

$$
\begin{aligned}
& \left.\frac{d W / W}{d t}\right|_{t=-\frac{1}{\sigma}}=\sigma(\gamma(\sigma-1)-1)+\sigma\left[1-\left(1-\frac{\lambda}{\sigma}\right)(1-\gamma)\right]+\left(1-\frac{\lambda}{\sigma}\right)(1-\gamma) \\
= & \sigma\left[\gamma(\sigma-1)-1+1-\left(1-\frac{\lambda}{\sigma}\right)(1-\gamma)\right]+\left(1-\frac{\lambda}{\sigma}\right)(1-\gamma) \\
= & \sigma \gamma(\sigma-1)-\sigma\left(1-\frac{\lambda}{\sigma}\right)(1-\gamma)+\left(1-\frac{\lambda}{\sigma}\right)(1-\gamma) \\
= & \sigma \gamma(\sigma-1)-\left(1-\frac{\lambda}{\sigma}\right)(1-\gamma)(\sigma-1) \\
\propto & \sigma \gamma-\left(1-\frac{\lambda}{\sigma}\right)(1-\gamma) .
\end{aligned}
$$

Hence, an optimal subsidy with a smaller rate than in the uniform policy case requires

$$
\begin{aligned}
\left.\frac{d W / W}{d t}\right|_{t=-\frac{1}{\sigma}} & >0 \Leftrightarrow \sigma \gamma>\left(1-\frac{\lambda}{\sigma}\right)(1-\gamma) \\
& \Leftrightarrow \frac{\gamma}{1-\gamma}>\frac{1}{\sigma}\left(1-\frac{\lambda}{\sigma}\right)
\end{aligned}
$$

Note that $\lambda$ is an endogenous object. A sufficient condition for the above inequality to hold is

$$
\frac{\gamma}{1-\gamma}>\frac{1}{\sigma} \Leftrightarrow \gamma \sigma>1-\gamma \Leftrightarrow \gamma>\frac{1}{\sigma+1} .
$$

If the labor cost share $\gamma$ is smaller, the optimal cooperative import subsidy rate is larger (in absolute values) than the uniform subsidy.

Let

$$
F(t, \gamma):=\gamma \frac{(1+t)\left(1-\left(\frac{\sigma \theta}{\sigma-1}-1\right) t \lambda\right)}{1+t-(1+t \lambda)(1-\gamma) \frac{\sigma-1}{\sigma}}-\left(\frac{\sigma \theta}{\sigma-1}-1\right) \frac{t \lambda}{\sigma-1}-1
$$

be the implicit function that determines optimal policy $t$.
Taking the derivative with respect to $\gamma$, we obtain

$$
\begin{aligned}
\frac{\partial F(t, \gamma)}{\partial \gamma} & =\frac{(1+t)\left(1-\left(\frac{\sigma \theta}{\sigma-1}-1\right) t \lambda\right)}{1+t-(1+t \lambda)(1-\gamma) \frac{\sigma-1}{\sigma}}-\frac{\gamma(1+t)\left(1-\left(\frac{\sigma \theta}{\sigma-1}-1\right) t \lambda\right)(1+t \lambda) \frac{\sigma-1}{\sigma}}{\left[1+t-(1+t \lambda)(1-\gamma) \frac{\sigma-1}{\sigma}\right]^{2}} \\
& =\frac{(1+t)\left(1-\left(\frac{\sigma \theta}{\sigma-1}-1\right) t \lambda\right)}{1+t-(1+t \lambda)(1-\gamma) \frac{\sigma-1}{\sigma}}\left(1-\frac{\gamma(1+t \lambda) \frac{\sigma-1}{\sigma}}{1+t-(1+t \lambda)(1-\gamma) \frac{\sigma-1}{\sigma}}\right) \\
& =\frac{(1+t)\left(1-\left(\frac{\sigma \theta}{\sigma-1}-1\right) t \lambda\right)}{1+t-(1+t \lambda)(1-\gamma) \frac{\sigma-1}{\sigma}} \frac{1+t-(1+t \lambda)(1-\gamma) \frac{\sigma-1}{\sigma}-\gamma(1+t \lambda) \frac{\sigma-1}{\sigma}}{1+t-(1+t \lambda)(1-\gamma) \frac{\sigma-1}{\sigma}} \\
& =\frac{(1+t)\left(1-\left(\frac{\sigma \theta}{\sigma-1}-1\right) t \lambda\right)}{1+t-(1+t \lambda)(1-\gamma) \frac{\sigma-1}{\sigma}} \times \frac{1+t-(1+t \lambda) \frac{\sigma-1}{\sigma}}{1+t-(1+t \lambda)(1-\gamma) \frac{\sigma-1}{\sigma}},
\end{aligned}
$$

where the first and the second term is positive for $t \leqslant 0$.
Taking the derivative with respect to $t$, we obtain

$$
\gamma \frac{(1+t)\left(1-\left(\frac{\sigma \theta}{\sigma-1}-1\right) t \lambda\right)}{1+t-(1+t \lambda)(1-\gamma) \frac{\sigma-1}{\sigma}}-\left(\frac{\sigma \theta}{\sigma-1}-1\right) \frac{t \lambda}{\sigma-1}-1
$$

$$
\begin{aligned}
\frac{\partial F(t, \gamma)}{\partial t}= & \gamma \frac{1-\left(\frac{\sigma \theta}{\sigma-1}-1\right) t \lambda-(1+t)\left(\frac{\sigma \theta}{\sigma-1}-1\right)\left(\lambda+t \frac{\partial \lambda}{\partial t}\right)}{1+t-(1+t \lambda)(1-\gamma) \frac{\sigma-1}{\sigma}} \\
& -\gamma \frac{(1+t)\left(1-\left(\frac{\sigma \theta}{\sigma-1}-1\right) t \lambda\right)\left(1-(1-\gamma) \frac{\sigma-1}{\sigma}\left(\lambda+t \frac{\partial \lambda}{\partial t}\right)\right)}{\left[1+t-(1+t \lambda)(1-\gamma) \frac{\sigma-1}{\sigma}\right]^{2}} \\
& -\left(\frac{\sigma \theta}{\sigma-1}-1\right) \frac{\lambda+t \frac{\partial \lambda}{\partial t}}{\sigma-1} .
\end{aligned}
$$

Evaluating this expression at $t=0$, we obtain

$$
\begin{aligned}
\left.\frac{\partial F(t, \gamma)}{\partial t}\right|_{t=0} & =\gamma \frac{1-\left(\frac{\sigma \theta}{\sigma-1}-1\right) \lambda}{1-(1-\gamma) \frac{\sigma-1}{\sigma}}-\gamma \frac{1-(1-\gamma) \frac{\sigma-1}{\sigma} \lambda}{\left(1-(1-\gamma) \frac{\sigma-1}{\sigma}\right)^{2}}-\left(\frac{\sigma \theta}{\sigma-1}-1\right) \frac{\lambda}{\sigma-1} \\
& =\frac{\gamma}{1-(1-\gamma) \frac{\sigma-1}{\sigma}}\left(1-\left(\frac{\sigma \theta}{\sigma-1}-1\right) \lambda-\frac{1-(1-\gamma) \frac{\sigma-1}{\sigma} \lambda}{1-(1-\gamma) \frac{\sigma-1}{\sigma}}\right)-\left(\frac{\sigma \theta}{\sigma-1}-1\right) \frac{\lambda}{\sigma-1} \\
& =\frac{\gamma}{1-(1-\gamma) \frac{\sigma-1}{\sigma}}\left(\frac{1-(1-\gamma) \frac{\sigma-1}{\sigma}-1+(1-\gamma) \frac{\sigma-1}{\sigma} \lambda}{1-(1-\gamma) \frac{\sigma-1}{\sigma}}-\left(\frac{\sigma \theta}{\sigma-1}-1\right) \lambda\right)-\left(\frac{\sigma \theta}{\sigma-1}-1\right) \\
& =\frac{\gamma}{1-(1-\gamma) \frac{\sigma-1}{\sigma}}\left(-\frac{(1-\gamma) \frac{\sigma-1}{\sigma}(1-\lambda)}{1-(1-\gamma) \frac{\sigma-1}{\sigma}}-\left(\frac{\sigma \theta}{\sigma-1}-1\right) \lambda\right)-\left(\frac{\sigma \theta}{\sigma-1}-1\right) \frac{\lambda}{\sigma-1}<0
\end{aligned}
$$

The implicit function theorem implies that

$$
\frac{\partial t}{\partial \gamma}=-\frac{\partial F(t, \gamma) / \partial \gamma}{\partial F(t, \gamma) / \partial t}
$$

Combining our previous observations, we have

$$
\left.\frac{\partial t}{\partial \gamma}\right|_{t=0}>0,
$$

which implies that starting from laissez faire, the optimal import subsidy is (in absolute values) increasing in $\gamma$.

We now take the derivative of $F$ with respect to $\lambda$, where the exogenous variation stems from $\eta$ :

$$
\frac{\partial F(t, \gamma)}{\partial \lambda}=\gamma(1+t)\left(\frac{\left(1-\left(\frac{\sigma \theta}{\sigma-1}-1\right) t \lambda\right) t \lambda(1-\gamma) \frac{\sigma-1}{\sigma} t}{\left[1+t-(1+t \lambda)(1-\gamma) \frac{\sigma-1}{\sigma}\right]^{2}}-\frac{\left(\frac{\sigma \theta}{\sigma-1}-1\right) t}{1+t-(1+t \lambda)(1-\gamma) \frac{\sigma-1}{\sigma}}\right)-\left(\frac{\sigma \theta}{\sigma-1}-1\right) \frac{1}{\sigma}
$$

Consider the limiting case $\gamma=0$. Then,

$$
\begin{aligned}
& \frac{\partial F(t, \gamma)}{\partial t}=-\left(\frac{\sigma \theta}{\sigma-1}-1\right) \lambda \frac{1+\frac{\partial \lambda}{\partial t} \frac{t}{\lambda}}{\sigma-1}<0 \text { and } \\
& \frac{\partial F(t, \gamma)}{\partial \lambda}=-\left(\frac{\sigma \theta}{\sigma-1}-1\right) \frac{\lambda}{\sigma-1}<0 .
\end{aligned}
$$

Note that $\lambda$ is decreasing in $\eta$. Then, by the implicit function theorem, we have $\partial t / \partial \eta>0$ for $\gamma=0$.

## A. 9 Non-cooperative trade policy

Income multiplier. The income multiplier is given by

$$
\begin{aligned}
\mu_{H} & =\frac{\left(1-(1-\gamma) \frac{\sigma-1}{\sigma}\right) \lambda_{H H}+\left(1+t_{F H}-(1-\gamma) \frac{\sigma-1}{\sigma}\right) \frac{1-\lambda_{H H}}{1+t_{F H}}}{\left(1-(1-\gamma) \frac{\sigma-1}{\sigma}\right)\left(\lambda_{H H}+\frac{1-\lambda_{H H}}{1+t_{F H}}\right)} \\
& =\frac{\left(1-(1-\gamma) \frac{\sigma-1}{\sigma}\right) \lambda_{H H}\left(1+t_{F H}\right)+\left(1+t_{F H}-(1-\gamma) \frac{\sigma-1}{\sigma}\right)\left(1-\lambda_{H H}\right)}{\left(1-(1-\gamma) \frac{\sigma-1}{\sigma}\right)\left(\lambda_{H H}\left(1+t_{F H}\right)+1-\lambda_{H H}\right)} \\
& =\frac{\lambda_{H H}\left(1+t_{F H}\right)-(1-\gamma) \frac{\sigma-1}{\sigma} \lambda_{H H}\left(1+t_{F H}\right)+\left(1+t_{F H}\right)\left(1-\lambda_{H H}\right)-(1-\gamma) \frac{\sigma-1}{\sigma}\left(1-\lambda_{H H}\right)}{\left(1-(1-\gamma) \frac{\sigma-1}{\sigma}\right)\left(\lambda_{H H}+\lambda_{H H} t_{F H}+1-\lambda_{H H}\right)} \\
& =\frac{-(1-\gamma) \frac{\sigma-1}{\sigma} \lambda_{H H}\left(1+t_{F H}\right)+1+t_{F H}-(1-\gamma) \frac{\sigma-1}{\sigma}\left(1-\lambda_{H H}\right)}{\left(1-(1-\gamma) \frac{\sigma-1}{\sigma}\right)\left(\lambda_{H H} t_{F H}+1\right)} \\
& =\frac{-(1-\gamma) \frac{\sigma-1}{\sigma} \lambda_{H H}-(1-\gamma) \frac{\sigma-1}{\sigma} \lambda_{H H} t_{F H}+1+t_{F H}-(1-\gamma) \frac{\sigma-1}{\sigma}+(1-\gamma) \frac{\sigma-1}{\sigma} \lambda_{H H}}{\left(1-(1-\gamma) \frac{\sigma-1}{\sigma}\right)\left(\lambda_{H H} t_{F H}+1\right)} \\
& =\frac{t_{F H}\left[1-(1-\gamma) \frac{\sigma-1}{\sigma} \lambda_{H H}\right]+1-(1-\gamma) \frac{\sigma-1}{\sigma}}{\left(1-(1-\gamma) \frac{\sigma-1}{\sigma}\right)\left(\lambda_{H H} t_{F H}+1\right)}
\end{aligned}
$$

Totally differentiating this expression, we obtain

$$
\begin{aligned}
\frac{d \mu_{H}}{\mu_{H}}= & \frac{\left[1-(1-\gamma) \frac{\sigma-1}{\sigma} \lambda_{H H}\right]\left(\lambda_{H H} t_{F H}+1\right)-\left(t_{F H}\left[1-(1-\gamma) \frac{\sigma-1}{\sigma} \lambda_{H H}\right]+\kappa^{-1}\right) \lambda_{H H}}{\left(\lambda_{H H} t_{F H}+1\right)\left(t_{F H}\left[1-(1-\gamma) \frac{\sigma-1}{\sigma} \lambda_{H H}\right]+\kappa^{-1}\right)} d t_{F H} \\
& -\left(\frac{\lambda_{H H} t_{F H}}{\lambda_{H H} t_{F H}+1}+\frac{(1-\gamma) \frac{\sigma-1}{\sigma} t_{F H} \lambda_{H H}}{t_{F H}\left[1-(1-\gamma) \frac{\sigma-1}{\sigma} \lambda_{H H}\right]+\kappa^{-1}}\right) \frac{d \lambda_{H H}}{\lambda_{H H}} \\
= & \left(\frac{1-(1-\gamma) \frac{\sigma-1}{\sigma} \lambda_{H H}}{t_{F H}\left[1-(1-\gamma) \frac{\sigma-1}{\sigma} \lambda_{H H}\right]+\kappa^{-1}}-\frac{\lambda_{H H}}{\lambda_{H H} t_{F H}+1}\right) d t_{F H} \\
& -\left(1+\frac{(1-\gamma) \frac{\sigma-1}{\sigma}}{\mu_{H}\left(1-(1-\gamma) \frac{\sigma-1}{\sigma}\right)}\right) \frac{\lambda_{H H} t_{F H}}{\lambda_{H H} t_{F H}+1} \frac{d \lambda_{H H}}{\lambda_{H H}} \\
= & \left(\frac{\left(1-(1-\gamma) \frac{\sigma-1}{\sigma} \lambda_{H H}\right)\left(\lambda_{H H} t_{F H}+1\right)}{t_{F H}\left[1-(1-\gamma) \frac{\sigma-1}{\sigma} \lambda_{H H}\right]+\kappa^{-1}}-\lambda_{H H}\right) \frac{d t_{F H}}{\lambda_{H H} t_{F H}+1} \\
& -\kappa \frac{\mu_{H}+(1-\gamma) \frac{\sigma-1}{\sigma}\left(1-\mu_{H}\right)}{\mu_{H}} \frac{\lambda_{H H} t_{F H}}{\lambda_{H H} t_{F H}+1} \frac{d \lambda_{H H}}{\lambda_{H H}} \\
= & \left(\kappa \frac{1-(1-\gamma) \frac{\sigma-1}{\sigma} \lambda_{H H}}{\mu_{H}}-\lambda_{H H}\right) \frac{d t_{F H}}{\lambda_{H H} t_{F H}+1} \\
& -\kappa\left(1-(1-\gamma) \frac{\sigma-1}{\sigma} \frac{\mu_{H}-1}{\mu_{H}}\right) \frac{\lambda_{H H} t_{F H}}{\lambda_{H H} t_{F H}+1} \frac{d \lambda_{H H}}{\lambda_{H H}} \\
= & \kappa \frac{1-(1-\gamma) \frac{\sigma-1}{\sigma} \lambda_{H H}-\lambda_{H H} \mu_{H}\left(1-(1-\gamma) \frac{\sigma-1}{\sigma}\right)}{\mu_{H}} \frac{d t_{F H}}{\lambda_{H H} t_{F H}+1} \\
& -\kappa\left(1-(1-\gamma) \frac{\sigma-1}{\sigma} \frac{\mu_{H}-1}{\mu_{H}}\right) \frac{\lambda_{H H} t_{F H}}{\lambda_{H H} t_{F H}+1} \frac{d \lambda_{H H}}{\lambda_{H H}} .
\end{aligned}
$$

Hence,

$$
\begin{aligned}
\frac{\lambda_{H H} t_{F H}+1}{\kappa} \frac{d \mu_{H}}{\mu_{H}}= & \frac{1-(1-\gamma) \frac{\sigma-1}{\sigma} \lambda_{H H}-\lambda_{H H} \mu_{H}\left(1-(1-\gamma) \frac{\sigma-1}{\sigma}\right)}{\mu_{H}} d t_{F H} \\
& -\left(1-(1-\gamma) \frac{\sigma-1}{\sigma} \frac{\mu_{H}-1}{\mu_{H}}\right) \lambda_{H H} t_{F H} \frac{d \lambda_{H H}}{\lambda_{H H}} \\
= & \frac{1-(1-\gamma) \frac{\sigma-1}{\sigma} \lambda_{H H}-\lambda_{H H} \mu_{H}+\lambda_{H H} \mu_{H}(1-\gamma) \frac{\sigma-1}{\sigma}}{\mu_{H}} d t_{F H} \\
& -\left(1-(1-\gamma) \frac{\sigma-1}{\sigma} \frac{\mu_{H}-1}{\mu_{H}}\right) \lambda_{H H} t_{F H} \frac{d \lambda_{H H}}{\lambda_{H H}} \\
= & \left(\frac{1}{\mu_{H}}-\lambda_{H H}+(1-\gamma) \frac{\sigma-1}{\sigma} \frac{\mu_{H}-1}{\mu_{H}} \lambda_{H H}\right) d t_{F H} \\
& -\left(1-(1-\gamma) \frac{\sigma-1}{\sigma} \frac{\mu_{H}-1}{\mu_{H}}\right) \lambda_{H H} t_{F H} \frac{d \lambda_{H H}}{\lambda_{H H}}
\end{aligned}
$$

Gross output multiplier. The gross output multiplier is given by

$$
\begin{aligned}
\tilde{\mu}_{H} & \equiv \frac{1}{\left(1-(1-\gamma) \frac{\sigma-1}{\sigma}\right)\left(\lambda_{H H}+\frac{1-\lambda_{H H}}{1+t_{F H}}\right)} \\
& =\frac{1+t_{F H}}{\left(1-(1-\gamma) \frac{\sigma-1}{\sigma}\right)\left(\left(1+t_{F H}\right) \lambda_{H H}+1-\lambda_{H H}\right)} \\
& =\frac{1+t_{F H}}{\left(1-(1-\gamma) \frac{\sigma-1}{\sigma}\right)\left(t_{F H} \lambda_{H H}+1\right)} .
\end{aligned}
$$

Totally differentiating this expression, we obtain

$$
\begin{aligned}
\frac{d \tilde{\mu}_{H}}{\mu_{H}} & =\left(\frac{1}{1+t_{F H}}-\frac{\lambda_{H H}}{t_{F H} \lambda_{H H}+1}\right) d t_{F H}-\frac{t_{F H} \lambda_{H H}}{t_{F H} \lambda_{H H}+1} \frac{d \lambda_{H H}}{\lambda_{H H}} \\
& =\left(\frac{t_{F H} \lambda_{H H}+1}{1+t_{F H}}-\lambda_{H H}\right) \frac{d t_{F H}}{t_{F H} \lambda_{H H}+1}-\frac{t_{F H} \lambda_{H H}}{t_{F H} \lambda_{H H}+1} \frac{d \lambda_{H H}}{\lambda_{H H}} \\
& =\left(\frac{t_{F H} \lambda_{H H}+1}{1+t_{F H}}-\lambda_{H H}\right) \frac{d t_{F H}}{t_{F H} \lambda_{H H}+1}-\frac{t_{F H} \lambda_{H H}}{t_{F H} \lambda_{H H}+1} \frac{d \lambda_{H H}}{\lambda_{H H}} \\
& =\left(t_{F H} \lambda_{H H}+1-\lambda_{H H}\left(1+t_{F H}\right)\right) \frac{1}{1+t_{F H}} \frac{d t_{F H}}{t_{F H} \lambda_{H H}+1}-\frac{t_{F H} \lambda_{H H}}{t_{F H} \lambda_{H H}+1} \frac{d \lambda_{H H}}{\lambda_{H H}} \\
& =\left(t_{F H} \lambda_{H H}+1-\lambda_{H H}-\lambda_{H H} t_{F H}\right) \frac{1}{1+t_{F H}} \frac{d t_{F H}}{t_{F H} \lambda_{H H}+1}-\frac{t_{F H} \lambda_{H H}}{t_{F H} \lambda_{H H}+1} \frac{d \lambda_{H H}}{\lambda_{H H}} \\
& =\frac{1-\lambda_{H H}}{t_{F H} \lambda_{H H}+1} \frac{d t_{F H}}{1+t_{F H}}-\frac{t_{F H} \lambda_{H H}}{t_{F H} \lambda_{H H}+1} \frac{d \lambda_{H H}}{\lambda_{H H}}
\end{aligned}
$$

Balanced trade (symmetric countries). With two symmetric countries, balanced trade reads

$$
\frac{w_{H}}{w_{F}}=\left(1+t_{F H}\right) \frac{1-\lambda_{F F}}{1-\lambda_{H H}} \frac{\tilde{\mu}_{F}}{\tilde{\mu}_{H}} .
$$

Totally differentiating this expression exploiting the facts that $t_{F F}=t_{H F}=0$, we obtain

$$
\begin{aligned}
\frac{d w_{H}}{w_{H} / w_{F}}= & \frac{1}{1+t_{F H}} d t_{F H}+\frac{\lambda_{H H}}{1-\lambda_{H H}} \frac{d \lambda_{H H}}{\lambda_{H H}}-\frac{\lambda_{F F}}{1-\lambda_{F F}} \frac{d \lambda_{F F}}{\lambda_{F F}}-\frac{d \tilde{\mu}_{H}}{\mu_{H}} \\
= & \frac{1}{1+t_{F H}} d t_{F H}+\frac{\lambda_{H H}}{1-\lambda_{H H}} \frac{d \lambda_{H H}}{\lambda_{H H}}-\frac{\lambda_{F F}}{1-\lambda_{F F}} \frac{d \lambda_{F F}}{\lambda_{F F}} \\
& -\frac{1-\lambda_{H H}}{t_{F H} \lambda_{H H}+1} \frac{d t_{F H}}{1+t_{F H}}+\frac{t_{F H} \lambda_{H H}}{t_{F H} \lambda_{H H}+1} \frac{d \lambda_{H H}}{\lambda_{H H}} \\
= & \left(1-\frac{1-\lambda_{H H}}{t_{F H} \lambda_{H H}+1}\right) \frac{d t_{F H}}{1+t_{F H}}+\left(1+\frac{t_{F H}\left(1-\lambda_{H H}\right)}{t_{F H} \lambda_{H H}+1}\right) \frac{\lambda_{H H}}{1-\lambda_{H H}} \frac{d \lambda_{H H}}{\lambda_{H H}}-\frac{\lambda_{F F}}{1-\lambda_{F F}} \frac{d \lambda_{F F}}{\lambda_{F F}} \\
= & \frac{t_{F H} \lambda_{H H}+1-1+\lambda_{H H}}{t_{F H} \lambda_{H H}+1} \frac{d t_{F H}}{1+t_{F H}}+\frac{t_{F H} \lambda_{H H}+1+t_{F H}-t_{F H} \lambda_{H H}}{t_{F H} \lambda_{H H}+1} \frac{\lambda_{H H}}{1-\lambda_{H H}} \frac{d \lambda_{H H}}{\lambda_{H H}} \\
& -\frac{\lambda_{F F}}{1-\lambda_{F F}} \frac{d \lambda_{F F}}{\lambda_{F F}} \\
= & \frac{t_{F H}+1}{t_{F H} \lambda_{H H}+1} \lambda_{H H} \frac{d t_{F H}}{1+t_{F H}}+\frac{1+t_{F H}}{t_{F H} \lambda_{H H}+1} \frac{\lambda_{H H}}{1-\lambda_{H H}} \frac{d \lambda_{H H}}{\lambda_{H H}}-\frac{\lambda_{F F}}{1-\lambda_{F F}} \frac{d \lambda_{F F}}{\lambda_{F F}} \\
= & \frac{\lambda_{H H}}{t_{F H} \lambda_{H H}+1} d t_{F H}+\frac{1+t_{F H}}{t_{F H} \lambda_{H H}+1} \frac{\lambda_{H H}}{1-\lambda_{H H}} \frac{d \lambda_{H H}}{\lambda_{H H}}-\frac{\lambda_{F F}}{1-\lambda_{F F}} \frac{d \lambda_{F F}}{\lambda_{F F}}
\end{aligned}
$$

Home's domestic expenditure share. With two symmetric countries, Home's domestic expenditure share is given by

$$
\lambda_{H H}=\frac{1}{1+\left(\frac{\tilde{\mu}_{F}}{\tilde{\mu}_{H}}\right)^{\frac{1-\gamma}{\gamma} \frac{\theta-(\sigma-1)}{\sigma-1}} \tau^{-\theta}\left(1+t_{F H}\right)^{1-\frac{\sigma \theta}{\sigma-1}}\left(\frac{f_{F H}}{f_{H H}}\right)^{\frac{\sigma-1-\theta}{\sigma-1}}\left(\frac{w_{H}}{w_{F}}\right)^{\frac{\theta \sigma}{\sigma-1}-1}\left(\frac{\lambda_{H H}}{\lambda_{F F}}\right)^{\frac{1-\gamma}{\gamma}}} .
$$

Totally differentiating this expression, we obtain

$$
\begin{aligned}
-\frac{1}{1-\lambda_{H H}} \frac{d \lambda_{H H}}{\lambda_{H H}}= & -\frac{1-\gamma}{\gamma} \frac{\theta-(\sigma-1)}{\sigma-1} \frac{d \tilde{\mu}_{H}}{\mu_{H}}+\left(1-\frac{\sigma \theta}{\sigma-1}\right) \frac{d t_{F H}}{1+t_{F H}}+\left(\frac{\theta \sigma}{\sigma-1}-1\right) \frac{d \frac{w_{H}}{w_{F}}}{w_{H} / w_{F}} \\
& +\frac{1-\gamma}{\gamma}\left(\frac{d \lambda_{H H}}{\lambda_{H H}}-\frac{d \lambda_{F F}}{\lambda_{F F}}\right) .
\end{aligned}
$$

Foreign expenditure share. By analogy, the change in Foreign's expenditure share is

$$
\begin{aligned}
-\frac{1}{1-\lambda_{F F}} \frac{d \lambda_{F F}}{\lambda_{F F}}= & \frac{1-\gamma}{\gamma} \frac{\theta-(\sigma-1)}{\sigma-1} \frac{d \tilde{\mu}_{H}}{\mu_{H}}+\left(1-\frac{\theta \sigma}{\sigma-1}\right) \frac{d \frac{w_{H}}{w_{F}}}{w_{H} / w_{F}} \\
& +\frac{1-\gamma}{\gamma}\left(\frac{d \lambda_{F F}}{\lambda_{F F}}-\frac{d \lambda_{H H}}{\lambda_{H H}}\right) \Leftrightarrow \\
-\left(\frac{1}{1-\lambda_{F F}}+\frac{1-\gamma}{\gamma}\right) \frac{d \lambda_{F F}}{\lambda_{F F}}= & \frac{1-\gamma}{\gamma} \frac{\theta-(\sigma-1)}{\sigma-1} \frac{d \tilde{\mu}_{H}}{\mu_{H}}+\left(1-\frac{\theta \sigma}{\sigma-1}\right) \frac{d \frac{w_{H}}{w_{F}}}{w_{H} / w_{F}}-\frac{1-\gamma}{\gamma \theta} \frac{d \lambda_{H H}}{\lambda_{H H}} .
\end{aligned}
$$

Using balanced trade to substitute out the change in the relative wage, we obtain

$$
\begin{aligned}
-\left(\frac{1}{1-\lambda_{F F}}+\frac{1-\gamma}{\gamma}\right) \frac{d \lambda_{F F}}{\lambda_{F F}}= & \frac{1-\gamma}{\gamma} \frac{\theta-(\sigma-1)}{\sigma-1} \frac{d \tilde{\mu}_{H}}{\mu_{H}}+\left(1-\frac{\theta \sigma}{\sigma-1}\right) \frac{\lambda_{H H}}{t_{F H} \lambda_{H H}+1} d t_{F H} \\
& -\frac{1-\gamma}{\gamma} \frac{d \lambda_{H H}}{\lambda_{H H}}+\left(1-\frac{\theta \sigma}{\sigma-1}\right) \frac{1+t_{F H}}{t_{F H} \lambda_{H H}+1} \frac{\lambda_{H H}}{1-\lambda_{H H}} \frac{d \lambda_{H H}}{\lambda_{H H}} \\
& -\left(1-\frac{\theta \sigma}{\sigma-1}\right) \frac{\lambda_{F F}}{1-\lambda_{F F}} \frac{d \lambda_{F F}}{\lambda_{F F}} .
\end{aligned}
$$

Collecting terms, we obtain

$$
\begin{aligned}
{\left[\frac{\left(1-\frac{\theta \sigma}{\sigma-1}\right) \lambda_{F F}-1}{1-\lambda_{F F}}-\frac{1-\gamma}{\gamma}\right] \frac{d \lambda_{F F}}{\lambda_{F F}}=} & \frac{1-\gamma}{\gamma} \frac{\theta-(\sigma-1)}{\sigma-1} \frac{d \tilde{\mu}_{H}}{\mu_{H}}+\frac{\left(1-\frac{\theta \sigma}{\sigma-1}\right) \lambda_{H H}}{t_{F H} \lambda_{H H}+1} d t_{F H} \\
& +\left(\frac{\left(1-\frac{\theta \sigma}{\sigma-1}\right)\left(1+t_{F H}\right)}{t_{F H} \lambda_{H H}+1} \frac{\lambda_{H H}}{1-\lambda_{H H}}-\frac{1-\gamma}{\gamma}\right) \frac{d \lambda_{H H}}{\lambda_{H H}}
\end{aligned}
$$

Hence,

$$
\frac{d \lambda_{F F}}{\lambda_{F F}}=\frac{\frac{1-\gamma}{\gamma} \frac{\theta-(\sigma-1)}{\sigma-1} \frac{d \tilde{\mu}_{H}}{\mu_{H}}+\frac{\left(1-\frac{\theta \sigma}{\sigma-1}\right) \lambda_{H H}}{t_{F H} \lambda_{H H}+1} d t_{F H}+\left(\frac{\left(1-\frac{\theta \sigma}{\sigma-1}\right)\left(1+t_{F H}\right)}{t_{F H} \lambda_{H H}+1} \frac{\lambda_{H H}}{1-\lambda_{H H}}-\frac{1-\gamma}{\gamma}\right) \frac{d \lambda_{H H}}{\lambda_{H H}}}{\frac{\left(1-\frac{\theta \sigma}{\sigma-1}\right) \lambda_{F F}-1}{1-\lambda_{F F}}-\frac{1-\gamma}{\gamma}}
$$

Home's domestic expenditure share, c'd. Substituting out the change in the relative wage, we obtain

$$
\begin{aligned}
\left(-\frac{1}{1-\lambda_{H H}}-\frac{1-\gamma}{\gamma}\right) \frac{d \lambda_{H H}}{\lambda_{H H}}= & -\frac{1-\gamma}{\gamma} \frac{\theta-(\sigma-1)}{\sigma-1} \frac{d \tilde{\mu}_{H}}{\mu_{H}} \\
& +\left(1-\frac{\sigma \theta}{\sigma-1}\right) \frac{d t_{F H}}{1+t_{F H}}+\left(\frac{\theta \sigma}{\sigma-1}-1\right) \frac{\lambda_{H H}}{t_{F H} \lambda_{H H}+1} d t_{F H} \\
& +\left(\frac{\theta \sigma}{\sigma-1}-1\right) \frac{1+t_{F H}}{t_{F H} \lambda_{H H}+1} \frac{\lambda_{H H}}{1-\lambda_{H H}} \frac{d \lambda_{H H}}{\lambda_{H H}} \\
& -\frac{1-\gamma}{\gamma} \frac{d \lambda_{F F}}{\lambda_{F F}}-\left(\frac{\theta \sigma}{\sigma-1}-1\right) \frac{\lambda_{F F}}{1-\lambda_{F F}} \frac{d \lambda_{F F}}{\lambda_{F F}}
\end{aligned}
$$

## Collecting terms, we obtain

$$
\begin{aligned}
& \left(-\frac{1}{1-\lambda_{H H}}-\frac{\left(1+t_{F H}\right)\left(\frac{\theta \sigma}{\sigma-1}-1\right)}{t_{F H} \lambda_{H H}+1} \frac{\lambda_{H H}}{1-\lambda_{H H}}-\frac{1-\gamma}{\gamma}\right) \frac{d \lambda_{H H}}{\lambda_{H H}} \\
=-\frac{1-\gamma}{\gamma} \frac{\theta-(\sigma-1)}{\sigma-1} \frac{d \tilde{\mu}_{H}}{\mu_{H}} & +\left(\frac{\theta \sigma}{\sigma-1}-1\right)\left(\frac{\lambda_{H H}}{t_{F H} \lambda_{H H}+1}-\frac{1}{1+t_{F H}}\right) d t_{F H} \\
& -\left(\frac{1-\gamma}{\gamma}+\frac{\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F}}{1-\lambda_{F F}}\right) \frac{d \lambda_{F F}}{\lambda_{F F}}
\end{aligned}
$$

where

$$
\left(\frac{\lambda_{H H}}{t_{F H} \lambda_{H H}+1}-\frac{1}{1+t_{F H}}\right) d t_{F H}=-\frac{1-\lambda_{H H}}{t_{F H} \lambda_{H H}+1} \frac{d t_{F H}}{1+t_{F H}}
$$

Hence,

$$
\begin{aligned}
& \left(-\frac{1}{1-\lambda_{H H}}-\frac{\left(1+t_{F H}\right)\left(\frac{\theta \sigma}{\sigma-1}-1\right)}{t_{F H} \lambda_{H H}+1} \frac{\lambda_{H H}}{1-\lambda_{H H}}-\frac{1-\gamma}{\gamma}\right) \frac{d \lambda_{H H}}{\lambda_{H H}} \\
=-\frac{1-\gamma}{\gamma} \frac{\theta-(\sigma-1)}{\sigma-1} \frac{d \tilde{\mu}_{H}}{\mu_{H}} & -\left(\frac{\theta \sigma}{\sigma-1}-1\right) \frac{1-\lambda_{H H}}{t_{F H} \lambda_{H H}+1} \frac{d t_{F H}}{1+t_{F H}} \\
& -\left(\frac{1-\gamma}{\gamma}+\frac{\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F}}{1-\lambda_{F F}}\right) \frac{d \lambda_{F F}}{\lambda_{F F}}
\end{aligned}
$$

Substituting out the change in Foreign's domestic expenditure share, we obtain

$$
\begin{aligned}
& \left(-\frac{1}{1-\lambda_{H H}}-\frac{\left(1+t_{F H}\right)\left(\frac{\theta \sigma}{\sigma-1}-1\right)}{t_{F H} \lambda_{H H}+1} \frac{\lambda_{H H}}{1-\lambda_{H H}}-\frac{1-\gamma}{\gamma}\right) \frac{d \lambda_{H H}}{\lambda_{H H}} \\
= & -\frac{\frac{1-\gamma}{\gamma}+\frac{\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F}}{1-\lambda_{F F}}}{\frac{\left(1-\frac{\theta \sigma}{\sigma-1}\right) \lambda_{F F}-1}{1-\lambda_{F F}}-\frac{1-\gamma}{\gamma}}\left(\frac{\left(1-\frac{\theta \sigma}{\sigma-1}\right)\left(1+t_{F H}\right)}{t_{F H} \lambda_{H H}+1} \frac{\lambda_{H H}}{1-\lambda_{H H}}-\frac{1-\gamma}{\gamma}\right) \frac{d \lambda_{H H}}{\lambda_{H H}} \\
& -\left(\frac{\theta \sigma}{\sigma-1}-1\right) \frac{1-\lambda_{H H}}{t_{F H} \lambda_{H H}+1} \frac{d t_{F H}}{1+t_{F H}}-\frac{\frac{1-\gamma}{\gamma}+\frac{\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F}}{1-\lambda_{F F}}}{\frac{\left(1-\frac{\theta \sigma}{\sigma-1}\right) \lambda_{F F}-1}{1-\lambda_{F F}}-\frac{1-\gamma}{\gamma}} \frac{\left(1-\frac{\theta \sigma}{\sigma-1}\right) \lambda_{H H}}{t_{F H} \lambda_{H H}+1} d t_{F H} \\
& -\frac{1-\gamma}{\gamma} \frac{\theta-(\sigma-1)}{\sigma-1} \frac{d \tilde{\mu}_{H}}{\mu_{H}}-\frac{1-\gamma}{\gamma}+\frac{\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F}}{1-\lambda_{F F}} \\
\frac{\left(1-\frac{\theta \sigma}{\sigma-1}\right) \lambda_{F F}-1}{1-\lambda_{F F}}-\frac{1-\gamma}{\gamma} & \frac{1-\gamma}{\gamma} \frac{\theta-(\sigma-1)}{\sigma-1} \frac{d \tilde{\mu}_{H}}{\mu_{H}}
\end{aligned}
$$

Collecting terms for the change in the domestic expenditure share, we obtain

$$
\begin{aligned}
& \left(-\frac{1}{1-\lambda_{H H}}-\frac{\left(1+t_{F H}\right)\left(\frac{\theta \sigma}{\sigma-1}-1\right)}{t_{F H} \lambda_{H H}+1} \frac{\lambda_{H H}}{1-\lambda_{H H}}-\frac{1-\gamma}{\gamma}+\frac{\frac{1-\gamma}{\gamma}+\frac{\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{A F}}{1-\lambda_{F F}}}{\frac{\left(1-\frac{\theta \sigma}{\sigma-1}\right) \lambda_{F F}-1}{1-\lambda_{F F}}-\frac{1-\gamma}{\gamma}}\left(\frac{\left(1-\frac{\theta \sigma}{\sigma-1}\right)\left(1+t_{F H}\right)}{t_{F H} \lambda_{H H}+1} \frac{\lambda_{H H}}{1-\lambda_{H H}}-\frac{1-\gamma}{\gamma}\right)\right) \frac{d \lambda_{H H}}{\lambda_{H H}} \\
& =\left(-\frac{1}{1-\lambda_{H H}}-\left(1+\frac{\frac{1-\gamma}{\gamma}+\frac{\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F}}{1-\lambda_{F F}}}{\frac{\left(1-\frac{\theta \sigma}{\sigma-1}\right) \lambda_{F F}-1}{1-\lambda_{F F}}-\frac{1-\gamma}{\gamma}}\right) \frac{\left(1+t_{F H}\right)\left(\frac{\theta \sigma}{\sigma-1}-1\right)}{t_{F H} \lambda_{H H}+1} \frac{\lambda_{H H}}{1-\lambda_{H H}}-\frac{1-\gamma}{\gamma}\left(1+\frac{\frac{1-\gamma}{\gamma}+\frac{\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F}}{1-\lambda_{F F}}}{\frac{\left(1-\frac{\theta \sigma}{\sigma-1}\right) \lambda_{F F}-1}{1-\lambda_{F F}}-\frac{1-\gamma}{\gamma}}\right)\right) \frac{d \lambda_{H H}}{\lambda_{H H}} \\
& =\left(-\frac{1}{1-\lambda_{H H}}-\frac{\frac{\left(1-\frac{\theta \sigma}{\sigma-1}\right) \lambda_{F F}-1}{1-\lambda_{F F}}-\frac{1-\gamma}{\gamma}+\frac{1-\gamma}{\gamma}+\frac{\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F}}{1-\lambda_{F F}}}{\frac{\left(1-\frac{\theta \sigma}{\sigma-1}\right) \lambda_{F F}-1}{1-\lambda_{F F}}-\frac{1-\gamma}{\gamma}} \frac{\left(1+t_{F H}\right)\left(\frac{\theta \sigma}{\sigma-1}-1\right)}{t_{F H} \lambda_{H H}+1} \frac{\lambda_{H H}}{1-\lambda_{H H}}-\frac{1-\gamma}{\gamma} \frac{\frac{\left(1-\frac{\theta \sigma}{\sigma-1}\right) \lambda_{F F}-1}{1-\lambda_{F F}}-\frac{1-\gamma}{\gamma}+\frac{1-\gamma}{\gamma}+\frac{\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F}}{1-\lambda_{F F}}}{\frac{\left(1-\frac{\theta \sigma}{\sigma-1}\right) \lambda_{F F}-1}{1-\lambda_{F F}}-\frac{1-\gamma}{\gamma}}\right) \frac{d \lambda_{H H}}{\lambda_{H H}} \\
& =\left(-\frac{1}{1-\lambda_{H H}}-\frac{\frac{\left(1-\frac{\theta \sigma}{\sigma-1}\right) \lambda_{F F}-1+\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F}}{1-\lambda_{A F}}}{\frac{\left(1-\frac{\theta \sigma}{\sigma-1}\right) \lambda_{F F}-1}{1-\lambda_{F F}}-\frac{1-\gamma}{\gamma}} \frac{\left(1+t_{F H}\right)\left(\frac{\theta \sigma}{\sigma-1}-1\right)}{t_{F H} \lambda_{H H}+1} \frac{\lambda_{H H}}{1-\lambda_{H H}}-\frac{1-\gamma}{\gamma} \frac{\frac{\left(1-\frac{\theta \sigma}{\sigma-1}\right) \lambda_{F F}-1+\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F}}{1-\lambda_{F F}}}{\frac{\left(1-\frac{\theta \sigma}{\sigma-1}\right) \lambda_{F F}-1}{1-\lambda_{F F}}-\frac{1-\gamma}{\gamma}}\right) \frac{d \lambda_{H H}}{\lambda_{H H}} \\
& =\left(-\frac{1}{1-\lambda_{H H}}+\frac{\frac{1}{1-\lambda_{F F}}}{\frac{\left(1-\frac{\theta \sigma}{\sigma-1}\right) \lambda_{F F}-1}{1-\lambda_{F F}}-\frac{1-\gamma}{\gamma}} \frac{\left(1+t_{F H}\right)\left(\frac{\theta \sigma}{\sigma-1}-1\right)}{t_{F H} \lambda_{H H}+1} \frac{\lambda_{H H}}{1-\lambda_{H H}}+\frac{1-\gamma}{\gamma} \frac{\frac{1}{1-\lambda_{F F}}}{\frac{\left(1-\frac{\theta \sigma}{\sigma-1}\right) \lambda_{F F}-1}{1-\lambda_{F F}}-\frac{1-\gamma}{\gamma}}\right) \frac{d \lambda_{H H}}{\lambda_{H H}} \\
& =\left(-\frac{1}{1-\lambda_{H H}}+\frac{1}{\left(1-\frac{\theta \sigma}{\sigma-1}\right) \lambda_{F F}-1-\frac{1-\gamma}{\gamma}\left(1-\lambda_{F F}\right)} \frac{\left(1+t_{F H}\right)\left(\frac{\theta \sigma}{\sigma-1}-1\right)}{t_{F H} \lambda_{H H}+1} \frac{\lambda_{H H}}{1-\lambda_{H H}}+\frac{1-\gamma}{\gamma} \frac{1}{\left(1-\frac{\theta \sigma}{\sigma-1}\right) \lambda_{F F}-1-\frac{1-\gamma}{\gamma}\left(1-\lambda_{F F}\right)}\right) \frac{d \lambda_{H H}}{\lambda_{H H}} \\
& =\frac{-\left(1-\frac{\theta \sigma}{\sigma-1}\right) \lambda_{F F}+1+\frac{1-\gamma}{\gamma}\left(1-\lambda_{F F}\right)+\frac{\left(1+t_{F H}\right)\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{H H}}{t_{F H} \lambda_{H H}+1}+\frac{1-\gamma}{\gamma}\left(1-\lambda_{H H}\right)}{\left(1-\frac{\theta \sigma}{\sigma-1}\right) \lambda_{F F}-1-\frac{1-\gamma}{\gamma}\left(1-\lambda_{F F}\right)} \frac{1}{1-\lambda_{H H}} \frac{d \lambda_{H H}}{\lambda_{H H}} \\
& =\frac{1+\left(\frac{\theta \sigma}{\sigma-1}-1\right)\left(\lambda_{F F}+\frac{1+t_{F H}}{t_{F H} \lambda_{H H}+1} \lambda_{H H}\right)+\frac{1-\gamma}{\gamma}\left(1-\lambda_{F F}\right)+\frac{1-\gamma}{\gamma}\left(1-\lambda_{H H}\right)}{\left(1-\frac{\theta \sigma}{\sigma-1}\right) \lambda_{F F}-1-\frac{1-\gamma}{\gamma}\left(1-\lambda_{F F}\right)} \frac{1}{1-\lambda_{H H}} \frac{d \lambda_{H H}}{\lambda_{H H}} \\
& =-\frac{1+\left(\frac{\theta \sigma}{\sigma-1}-1\right)\left(\lambda_{F F}+\frac{1+t_{F H}}{t_{F H} \lambda_{H H}+1} \lambda_{H H}\right)+\frac{1-\gamma}{\gamma}\left(2-\lambda_{H H}-\lambda_{F F}\right)}{\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F}+1+\frac{1-\gamma}{\gamma}\left(1-\lambda_{F F}\right)} \frac{1}{1-\lambda_{H H}} \frac{d \lambda_{H H}}{\lambda_{H H}}
\end{aligned}
$$

Collecting terms for the direct tariff effect, we have

$$
\left(-\frac{\frac{1-\gamma}{\gamma}+\frac{\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F}}{1-\lambda_{F F}}}{\frac{1-\gamma}{\gamma}+\frac{\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F}+1}{1-\lambda_{F F}}} \lambda_{H H}-\frac{1-\lambda_{H H}}{1+t_{F H}}\right) \frac{\left(\frac{\theta \sigma}{\sigma-1}-1\right)}{t_{F H} \lambda_{H H}+1} d t_{F H}
$$

and

$$
\begin{aligned}
& -\frac{1+\left(\frac{\theta \sigma}{\sigma-1}-1\right)\left(\lambda_{F F}+\mu_{H} \lambda_{H H}\right)}{\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F}+1} \frac{1}{1-\lambda_{H H}} \frac{d \lambda_{H H}}{\lambda_{H H}} \\
& =\left(-\frac{\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F}}{\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F}+1} \lambda_{H H}-\frac{1-\lambda_{H H}}{1+t_{F H}}\right) \frac{\left(\frac{\theta \sigma}{\sigma-1}-1\right)}{t_{F H} \lambda_{H H}+1} d t_{F H} \Leftrightarrow \\
& \frac{1+\left(\frac{\theta \sigma}{\sigma-1}-1\right)\left(\lambda_{F F}+\mu_{H} \lambda_{H H}\right)}{1-\lambda_{H H}} \frac{d \lambda_{H H}}{\lambda_{H H}} \\
& =\left(\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F} \lambda_{H H}+\frac{1-\lambda_{H H}}{1+t_{F H}}\left(\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F}+1\right)\right) \frac{\left(\frac{\theta \sigma}{\sigma-1}-1\right)}{t_{F H} \lambda_{H H}+1} d t_{F H} \\
& =\left(\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F} \lambda_{H H}+\frac{1-\lambda_{H H}}{1+t_{F H}}\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F}+\frac{1-\lambda_{H H}}{1+t_{F H}}\right) \frac{\left(\frac{\theta \sigma}{\sigma-1}-1\right)}{t_{F H} \lambda_{H H}+1} d t_{F H} \\
& =\left(\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F}\left(\lambda_{H H}+\frac{1-\lambda_{H H}}{1+t_{F H}}\right)+\frac{1-\lambda_{H H}}{1+t_{F H}}\right) \frac{\left(\frac{\theta \sigma}{\sigma-1}-1\right)}{t_{F H} \lambda_{H H}+1} d t_{F H} \\
& =\left(\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F} \frac{\lambda_{H H}+\lambda_{H H} t_{F H}+1-\lambda_{H H}}{1+t_{F H}}+\frac{1-\lambda_{H H}}{1+t_{F H}}\right) \frac{\left(\frac{\theta \sigma}{\sigma-1}-1\right)}{t_{F H} \lambda_{H H}+1} d t_{F H} \\
& =\left(\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F} \frac{\lambda_{H H} t_{F H}+1}{1+t_{F H}}+\frac{1-\lambda_{H H}}{1+t_{F H}}\right) \frac{\left(\frac{\theta \sigma}{\sigma-1}-1\right)}{t_{F H} \lambda_{H H}+1} d t_{F H} \\
& =\left(\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F}+\frac{1-\lambda_{H H}}{t_{F H} \lambda_{H H}+1}\right)\left(\frac{\theta \sigma}{\sigma-1}-1\right) \frac{d t_{F H}}{1+t_{F H}} \\
& =\left(1-\lambda_{H H} \mu_{H}+\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F}+\frac{1-\lambda_{H H}}{t_{F H} \lambda_{H H}+1}-1+\lambda_{H H} \mu_{H}\right)\left(\frac{\theta \sigma}{\sigma-1}-1\right) \frac{d t_{F H}}{1+t_{F H}} \\
& =\left(1-\lambda_{H H} \mu_{H}+\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F}+\frac{1-\lambda_{H H}-t_{F H} \lambda_{H H}-1}{t_{F H} \lambda_{H H}+1}+\lambda_{H H} \mu_{H}\right)\left(\frac{\theta \sigma}{\sigma-1}-1\right) \frac{d t_{F H}}{1+t_{F H}} \\
& =\left(1-\lambda_{H H} \mu_{H}+\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F}-\frac{1+t_{F H}}{t_{F H} \lambda_{H H}+1} \lambda_{H H}+\lambda_{H H} \mu_{H}\right)\left(\frac{\theta \sigma}{\sigma-1}-1\right) \frac{d t_{F H}}{1+t_{F H}} \\
& =\left(1-\lambda_{H H} \mu_{H}+\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F}-\mu_{H} \lambda_{H H}+\lambda_{H H} \mu_{H}\right)\left(\frac{\theta \sigma}{\sigma-1}-1\right) \frac{d t_{F H}}{1+t_{F H}} \\
& =\left(1-\lambda_{H H} \mu_{H}+\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F}\right)\left(\frac{\theta \sigma}{\sigma-1}-1\right) \frac{d t_{F H}}{1+t_{F H}}
\end{aligned}
$$

Collecting terms for the gross output multiplier, we obtain

$$
\begin{aligned}
& -\frac{1-\gamma}{\gamma} \frac{\theta-(\sigma-1)}{\sigma-1} \frac{d \tilde{\mu}_{H}}{\mu_{H}}-\frac{\frac{1-\gamma}{\gamma}+\frac{\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F}}{1-\lambda_{F F}}}{\frac{\left(1-\frac{\theta \sigma}{\sigma-1}\right) \lambda_{F F}-1}{1-\lambda_{F F}}-\frac{1-\gamma}{\gamma}} \frac{1-\gamma}{\gamma} \frac{\theta-(\sigma-1)}{\sigma-1} \frac{d \tilde{\mu}_{H}}{\mu_{H}} \\
= & -\left(1+\frac{\frac{1-\gamma}{\gamma}+\frac{\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F}}{1-\lambda_{F F}}}{\frac{\left(1-\frac{\theta \sigma}{\sigma-1}\right) \lambda_{F F}-1}{1-\lambda_{F F}}-\frac{1-\gamma}{\gamma}}\right) \frac{1-\gamma}{\gamma} \frac{\theta-(\sigma-1)}{\sigma-1} \frac{d \tilde{\mu}_{H}}{\mu_{H}} \\
= & -\frac{\frac{\left(1-\frac{\theta \sigma}{\sigma-1}\right) \lambda_{F F}-1}{1-\lambda_{F F}}-\frac{1-\gamma}{\gamma}+\frac{1-\gamma}{\gamma}+\frac{\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F}}{1-\lambda_{F F}}}{\frac{\left(1-\frac{\theta \sigma}{\sigma-1}\right) \lambda_{F F}-1}{1-\lambda_{F F}}-\frac{1-\gamma}{\gamma}} \frac{1-\gamma}{\gamma} \frac{\theta-(\sigma-1)}{\sigma-1} \frac{d \tilde{\mu}_{H}}{\mu_{H}} \\
= & -\frac{\frac{\left(1-\frac{\theta \sigma}{\sigma-1}\right) \lambda_{F F}-1+\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F}}{1-\lambda_{\lambda_{F}}}}{\frac{\left(1-\frac{\theta \sigma}{\sigma-1}\right) \lambda_{F F}-1}{1-\lambda_{F F}}-\frac{1-\gamma}{\gamma}} \frac{1-\gamma}{\gamma} \frac{\theta-(\sigma-1)}{\sigma-1} \frac{d \tilde{\mu}_{H}}{\mu_{H}} \\
= & \frac{1}{\left(1-\frac{\theta \sigma}{\sigma-1}\right) \lambda_{F F}-1-\frac{1-\gamma}{\gamma}\left(1-\lambda_{F F}\right)} \frac{1-\gamma}{\gamma} \frac{\theta-(\sigma-1)}{\sigma-1} \frac{d \tilde{\mu}_{H}}{\mu_{H}} \\
= & -\frac{1}{\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F}+1+\frac{1-\gamma}{\gamma}\left(1-\lambda_{F F}\right)} \frac{1-\gamma}{\gamma} \frac{\theta-(\sigma-1)}{\sigma-1} \frac{d \tilde{\mu}_{H}}{\mu_{H}}
\end{aligned}
$$

Combining direct and gross output multiplier effects

$$
\begin{aligned}
&\left(-\frac{\frac{1-\gamma}{\gamma}+\frac{\left(\frac{\theta \sigma}{\sigma-1}-1\right)_{A_{F F}}}{1-\lambda_{F F}}}{\frac{1-\gamma}{\gamma}+\frac{\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F}+1}{1-\lambda_{F F}}} \lambda_{H H}-\frac{1-\lambda_{H H}}{1+t_{F H}}\right) \frac{\left(\frac{\theta \sigma}{\sigma-1}-1\right)}{t_{F H} \lambda_{H H}+1} d t_{F H} \\
&-\frac{1}{\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F}+1+\frac{1-\gamma}{\gamma}\left(1-\lambda_{F F}\right)} \frac{1-\gamma}{\gamma} \frac{\theta-(\sigma-1)}{\sigma-1} \frac{d \tilde{\mu}_{H}}{\mu_{H}} \\
&=\left(-\frac{\left.\frac{1-\gamma}{\gamma}+\frac{\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F}}{\frac{1-\gamma}{\gamma}+\frac{\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F}+1}{1-\lambda_{F F}}} \lambda_{H H}-\frac{1-\lambda_{H H}}{1+t_{F H}}\right) \frac{\left(\frac{\theta \sigma}{\sigma-1}-1\right)}{t_{F H} \lambda_{H H}+1} d t_{F H}}{}\right. \\
&-\frac{1}{\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F}+1+\frac{1-\gamma}{\gamma}\left(1-\lambda_{F F}\right)} \frac{1-\gamma}{\gamma} \frac{\theta-(\sigma-1)}{\sigma-1} \frac{1-\lambda_{H H}}{t_{F H} \lambda_{H H}+1} \frac{d t_{F H}}{1+t_{F H}} \\
&+\frac{1}{\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F}+1+\frac{1-\gamma}{\gamma}\left(1-\lambda_{F F}\right)} \frac{1-\gamma}{\gamma} \frac{\theta-(\sigma-1)}{\sigma-1} \frac{t_{F H} \lambda_{H H}}{t_{F H} \lambda_{H H}+1} \frac{d \lambda_{H H}}{\lambda_{H H}}
\end{aligned}
$$

Collecting direct effects

$$
\begin{array}{rl} 
& \left(-\frac{\frac{1-\gamma}{\gamma}+\frac{\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F}}{1-\lambda_{F F}}}{\frac{1-\gamma}{\gamma}+\frac{\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F}+1}{1-\lambda_{F F}}} \lambda_{H H}-\frac{1-\lambda_{H H}}{1+t_{F H}}\right) \frac{\left(\frac{\theta \sigma}{\sigma-1}-1\right)}{t_{F H} \lambda_{H H}+1} d t_{F H} \\
& -\frac{1}{\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F}+1+\frac{1-\gamma}{\gamma}\left(1-\lambda_{F F}\right)} \frac{1-\gamma}{\gamma} \frac{\theta-(\sigma-1)}{\sigma-1} \frac{1-\lambda_{H H}}{t_{F H} \lambda_{H H}+1} \frac{d t_{F H}}{1+t_{F H}} \\
= & \left(-\frac{\frac{1-\gamma}{\gamma}+\frac{\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F}}{1-\lambda_{F F}}}{\frac{1-\gamma}{\gamma}+\frac{\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F}+1}{1-\lambda_{F F}}} \lambda_{H H}-\frac{1-\lambda_{H H}}{1+t_{F H}}\right)\left(\frac{\theta \sigma}{\sigma-1}-1\right) \frac{d t_{F H}}{t_{F H} \lambda_{H H}+1} \\
= & \left(-\frac{\frac{1-\gamma}{\gamma} \frac{\theta-(\sigma-1)}{\sigma-1}}{\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F}+1+\frac{1-\gamma}{\gamma \theta}\left(1-\lambda_{F F}\right)} \frac{1-\lambda_{H H}}{1+t_{F H}} \frac{d t_{F H}}{t_{F H} \lambda_{H H}+1}\right. \\
= & \left(\frac{1-\gamma}{\gamma}+\frac{\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F}}{1-\lambda_{F F}}\right. \\
& \left.-\frac{\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F}+1}{1-\lambda_{F F}} \frac{\lambda_{H H}\left(1+t_{F H}\right)}{1-\lambda_{H H}}-1\right)\left(\frac{\theta \sigma}{\sigma-1}-1\right) \frac{d t_{F H}}{t_{F H} \lambda_{H H}+1} \\
\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F}+1+\frac{1-\gamma}{\gamma}\left(1-\lambda_{F F}\right) & 1-\lambda_{H H} \\
1+t_{F H} & d t_{F H} \\
t_{F H} \lambda_{H H}+1
\end{array}
$$

Collecting domestic expenditure share terms, we obtain

$$
\begin{aligned}
& -\frac{1+\left(\frac{\theta \sigma}{\sigma-1}-1\right)\left(\lambda_{F F}+\frac{1+t_{F H}}{t_{F H} \lambda_{F H}} \lambda_{H H}\right)+\frac{1-\gamma}{\gamma}\left(2-\lambda_{H H}-\lambda_{F F}\right)}{\left(\frac{\theta_{\sigma}}{\sigma-1}-1\right) \lambda_{F F}+1+\frac{1-\gamma}{\gamma}\left(1-\lambda_{F F}\right)} \frac{1}{1-\lambda_{H H}} \frac{d \lambda_{H H}}{\lambda_{H H}} \\
& -\frac{\frac{1-\gamma}{\gamma} \frac{\theta-(\sigma-1)}{\sigma-1}}{\left(\frac{\theta_{\sigma}}{\sigma-1}-1\right) \lambda_{F F}+1+\frac{1-\gamma}{\gamma}\left(1-\lambda_{F F}\right)} \frac{t_{F H} \lambda_{H H}}{t_{F H} \lambda_{H H}+1} \frac{d \lambda_{H H}}{\lambda_{H H}} \\
& -\frac{1+\left(\frac{\theta \sigma}{\sigma-1}-1\right)\left(\lambda_{F F}+\frac{1+t_{F H}}{t_{F H} \lambda_{H H}+1} \lambda_{H H}\right)+\frac{1-\gamma}{\gamma}\left(2-\lambda_{H H}-\lambda_{F F}\right)+\frac{1-\gamma}{\gamma} \frac{\theta-(\sigma-1)}{\sigma-1} \frac{t_{F H} \lambda_{H H}\left(1-\lambda_{H H}\right)}{t_{F H} \lambda_{H H}+1}}{\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F}+1+\frac{1-\gamma}{\gamma}\left(1-\lambda_{F F}\right)} \frac{1}{1-\lambda_{H H}} \frac{d \lambda_{H H}}{\lambda_{H H}}
\end{aligned}
$$

Hence, the tariff-induced change in the domestic expenditure share is determined by

$$
\begin{aligned}
& -\frac{1+\left(\frac{\theta \sigma}{\sigma-1}-1\right)\left(\lambda_{F F}+\frac{1+t_{F H}}{t_{F H} \lambda_{H H}+1} \lambda_{H H}\right)+\frac{1-\gamma}{\gamma}\left(2-\lambda_{H H}-\lambda_{F F}\right)+\frac{1-\gamma}{\gamma} \frac{\theta-(\sigma-1)}{\sigma-1} \frac{t_{F H} \lambda_{H H}\left(1-\lambda_{H H}\right)}{t_{F H} \lambda_{H H}+1}}{\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F}+1+\frac{1-\gamma}{\gamma}\left(1-\lambda_{F F}\right)} \frac{1}{1-\lambda_{H H}} \frac{d \lambda_{H H}}{\lambda_{H H}} \\
= & {\left[\left(-\frac{\frac{1-\gamma}{\gamma}+\frac{\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F}}{1-\lambda_{F F}}}{\frac{1-\gamma}{\gamma}+\frac{\left(\frac{\theta \sigma}{\sigma-1} 1\right) \lambda_{F F}+1}{1-\lambda_{F F}}} \frac{\lambda_{H H}\left(1+t_{F H}\right)}{1-\lambda_{H H}}-1\right)\left(\frac{\theta \sigma}{\sigma-1}-1\right)-\frac{\frac{1-\gamma}{\gamma} \frac{\theta-(\sigma-1)}{\sigma-1}}{\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F}+1+\frac{1-\gamma}{\gamma}\left(1-\lambda_{F F}\right)}\right] \frac{1-\lambda_{H H}}{1+t_{F H}} \frac{d t_{F H}}{t_{F H} \lambda_{H H}+1} }
\end{aligned}
$$

Hence,

$$
\begin{aligned}
& \frac{d \lambda_{H H}}{\lambda_{H H}}=-\frac{\left(-\frac{\frac{1-\gamma}{\gamma}+\frac{\left(\frac{\theta \sigma}{\sigma}-1\right) \lambda_{F F}}{1-\lambda_{F F}}}{\frac{1-\gamma}{\gamma}+\frac{\left(\frac{\theta \sigma}{\sigma-1}-1 \lambda_{F F}+1\right.}{1-\lambda_{F F}}} \frac{\lambda_{H H}\left(1+t_{F H}\right)}{1-\lambda_{H H}}-1\right)\left(\frac{\theta \sigma}{\sigma-1}-1\right)-\frac{\frac{1-\gamma}{\gamma} \frac{\theta-(\sigma-1)}{\sigma-1}}{\left(\frac{\theta \sigma}{\sigma \sigma-1}-1\right) \lambda_{F F}+1+\frac{1-\gamma}{\gamma}\left(1-\lambda_{F F}\right)}}{\frac{1+\left(\frac{\theta \sigma}{\sigma-1}-1\right)\left(\lambda_{F F}+\frac{1+t_{F H}}{t_{F H} \lambda_{H H}+1} \lambda_{H H}\right)+\frac{1-\gamma}{\gamma}\left(2-\lambda_{H H}-\lambda_{F F}\right)+\frac{1-\gamma}{\gamma} \frac{\theta-(\sigma-1)}{\sigma-1} \frac{t_{F H} \lambda_{H H}\left(1-\lambda_{H H}\right)}{t_{F H} \lambda_{H H}+1}}{\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F}+1+\frac{1-\gamma}{\gamma}\left(1-\lambda_{F F}\right)}} \frac{\left(1-\lambda_{H H}\right)^{2}}{t_{F H} \lambda_{H H}+1} \frac{d t_{F H}}{1+t_{F H}} \\
& =\frac{\left(\frac{\frac{1-\gamma}{\gamma}+\frac{\left(\frac{\theta \sigma}{\sigma}-1\right) \lambda_{F F}}{\sigma-\lambda_{F F}}}{\frac{1-\gamma}{\gamma}+\frac{\left(\frac{\theta \sigma}{\sigma-1}-1 \lambda_{F F}+1\right.}{1-\lambda_{F F}}} \frac{\lambda_{H H}\left(1+t_{F H}\right)}{1-\lambda_{H H}}+1\right)\left(\frac{\theta \sigma}{\sigma-1}-1\right)+\frac{\frac{1-\gamma}{\gamma} \frac{\theta-(\sigma-1)}{\sigma-1}}{\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F}+1+\frac{1-\gamma}{\gamma}\left(1-\lambda_{F F}\right)}}{\frac{1+\left(\frac{\theta \sigma}{\sigma-1}-1\right)\left(\lambda_{F F}+\frac{1+t_{F H}}{t_{F H} \lambda_{H H}+1} \lambda_{H H}\right)+\frac{1-\gamma}{\gamma}\left(2-\lambda_{H H}-\lambda_{F F}\right)+\frac{1-\gamma}{\gamma} \frac{\theta-(\sigma-1)}{\sigma-1} \frac{t_{F H} \lambda_{H H( }\left(1-\lambda_{H H}\right)}{t_{F H} \lambda_{H H}+1}}{\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F}+1+\frac{1-\gamma}{\gamma}\left(1-\lambda_{F F}\right)}} \frac{\left(1-\lambda_{H H}\right)^{2}}{t_{F H} \lambda_{H H}+1} \frac{d t_{F H}}{1+t_{F H}}
\end{aligned}
$$

Rearranging terms, we obtain

$$
\begin{aligned}
& \frac{d \lambda_{H H}}{\lambda_{H H}}=\frac{\left(\frac{\frac{1-\gamma}{\gamma}+\frac{\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F}}{\sigma-\lambda_{F F}}}{\frac{1-\gamma}{\gamma}+\frac{\left(\frac{\theta}{\sigma-1}-1\right)_{F F}+1}{\sigma-1}} \frac{\lambda_{H H}\left(1+t_{F H}\right)}{1-\lambda_{F F}}+1\right)\left(\frac{\theta \sigma}{\sigma-1}-1\right)\left[\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F}+1+\frac{1-\gamma}{\gamma}\left(1-\lambda_{F F}\right)\right]+\frac{1-\gamma}{\gamma} \frac{\theta-(\sigma-1)}{\sigma-1}}{1+\left(\frac{\theta \sigma}{\sigma-1}-1\right)\left(\lambda_{F F}+\frac{1+t_{F H}}{t_{F H} \lambda_{H H}+1} \lambda_{H H}\right)+\frac{1-\gamma}{\gamma}\left(2-\lambda_{H H}-\lambda_{F F}\right)+\frac{1-\gamma}{\gamma} \frac{\theta-(\sigma-1)}{\sigma-1} \frac{t_{F H} \lambda_{H H}\left(1-\lambda_{H H}\right)}{t_{F H} \lambda_{H H}+1}} \frac{\left(1-\lambda_{H H}\right)^{2}}{t_{F H} \lambda_{H H}+1} \frac{d t_{F H}}{1+t_{F H}} \\
& =\frac{\left(\frac{\frac{1-\gamma}{}\left(1-\lambda_{F F}\right)+\left(\frac{\partial \sigma}{\sigma \sigma}-1\right) \lambda_{F F}}{\frac{1-\gamma}{\gamma}\left(1-\lambda_{F F}\right)+\left(\frac{\sigma^{\sigma}}{\sigma-1}-1\right) \lambda_{F F}+1} \frac{\lambda_{H H}\left(1+t_{F H}\right)}{1-\lambda_{H H}}+1\right)\left(\frac{\theta \sigma}{\sigma-1}-1\right)\left[\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F}+1+\frac{1-\gamma}{\gamma}\left(1-\lambda_{F F}\right)\right]+\frac{1-\gamma}{\gamma} \frac{\theta-(\sigma-1)}{\sigma-1}}{1+\left(\frac{\theta \sigma}{\sigma-1}-1\right)\left(\lambda_{F F}+\frac{1+t_{F H}}{t_{F H} \lambda_{H H}+1} \lambda_{H H}\right)+\frac{1-\gamma}{\gamma}\left(2-\lambda_{H H}-\lambda_{F F}+\frac{\theta-(\sigma-1)}{\sigma-1} \frac{t_{F H} \lambda_{H H}\left(1-\lambda_{H H}\right)}{t_{F H} \lambda_{H H}+1}\right)} \frac{\left.\lambda_{H H}\right)^{2}}{t_{F H} \lambda_{H H}+1} \frac{d t_{F H}}{1+t_{F H}} \\
& =\frac{\left(\left(\frac{1-\gamma}{\gamma}\left(1-\lambda_{F F}\right)+\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F}\right) \frac{\lambda_{H H}\left(1+t_{F H}\right)}{1-\lambda_{H H}}+\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F}+1+\frac{1-\gamma}{\gamma}\left(1-\lambda_{F F}\right)\right)\left(\frac{\theta \sigma}{\sigma-1}-1\right)+\frac{1-\gamma}{\gamma} \frac{\theta-(\sigma-1)}{\sigma-1}}{1+\left(\frac{\theta \sigma}{\sigma-1}-1\right)\left(\lambda_{F F}+\frac{1+t_{F H}}{t_{F H} \lambda_{H H}+1} \lambda_{H H}\right)+\frac{1-\gamma}{\gamma}\left(2-\lambda_{H H}-\lambda_{F F}+\frac{\theta-(\sigma-1)}{\sigma-1} \frac{t_{F H} \lambda_{H H}\left(1-\lambda_{H H}\right)}{t_{F H} \lambda_{H H}+1}\right)} \frac{\left.\lambda_{H H}\right)^{2}}{t_{F H} \lambda_{H H}+1} \frac{d t_{F H}}{1+t_{F H}} \\
& =\frac{\left(\frac{1-\gamma}{\gamma}\left(1-\lambda_{F F}\right)+\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F}\right) \frac{\lambda_{H H}\left(1+t_{F H}\right)}{1-\lambda_{H H}}+\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F}+1+\frac{1-\gamma}{\gamma}\left(1-\lambda_{F F}\right)+\frac{1-\gamma}{\gamma} \frac{\frac{\theta-(\sigma-1)}{\sigma-1}}{\frac{\sigma-1}{\sigma-1}-1}}{1+\left(\frac{\theta \sigma}{\sigma-1}-1\right)\left(\lambda_{F F}+\frac{1+t_{F H}}{t_{F H} \lambda_{H H}+1} \lambda_{H H}\right)+\frac{1-\gamma}{\gamma}\left(2-\lambda_{H H}-\lambda_{F F}+\frac{\theta-(\sigma-1)}{\sigma-1} \frac{t_{F H} \lambda_{H H}\left(1-\lambda_{H H}\right.}{t_{F H} \lambda_{H H}+1}\right)} \frac{\theta \sigma}{t_{F H} \lambda_{H H}+1}\left(\frac{\theta \sigma}{\sigma-1}-1\right) \frac{d t_{F H}}{1+t_{F H}} \\
& =\frac{\frac{1-\gamma}{\gamma}\left(1-\lambda_{F F}\right)\left(\frac{\lambda_{H H}\left(1+t_{F H}\right)}{1-\lambda_{H H}}+1\right)+\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F}\left(\frac{\lambda_{H H}\left(1+t_{F H}\right)}{1-\lambda_{H H}}+1\right)+1+\frac{1-\gamma}{\gamma} \frac{\frac{\theta-(\sigma-1)}{\sigma-1}}{\sigma \sigma-1}}{1+\left(\frac{\theta \sigma}{\sigma-1}-1\right)\left(\lambda_{F F}+\frac{1+t_{F H}}{t_{F H} \lambda_{H H}+1} \lambda_{H H}\right)+\frac{1-\gamma}{\gamma}\left(2-\lambda_{H H}-\lambda_{F F}+\frac{\theta-(\sigma-1)}{\sigma-1} \frac{t_{F H} \lambda_{H H}\left(1-\lambda_{H H}\right)}{t_{F H} \lambda_{H H}+1}\right)} \frac{\left(1-\lambda_{H H}\right)^{2}}{t_{F H} \lambda_{H H}+1}\left(\frac{\theta \sigma}{\sigma-1}-1\right) \frac{d t_{F H}}{1+t_{F H}} \\
& =\frac{\frac{1-\gamma}{\gamma}\left(1-\lambda_{F F}\right)\left(\frac{\lambda_{H H}\left(1+t_{F H}\right)}{1-\lambda_{H H}}+1\right)+\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F}\left(\frac{\lambda_{H H}\left(1+t_{F H}\right)}{1-\lambda_{H H}}+1\right)+1+\frac{1-\gamma}{\gamma} \frac{\frac{\theta-(\sigma-1)}{\sigma-1}}{\sigma \sigma}}{1+\left(\frac{\theta \sigma}{\sigma-1}-1\right)\left(\lambda_{F F}+\frac{1+t_{F H}}{t_{F H} \lambda_{H H}+1} \lambda_{H H}\right)+\frac{1-\gamma}{\gamma}\left(2-\lambda_{H H}-\lambda_{F F}+\frac{\theta-(\sigma-1)}{\sigma-1} \frac{t_{F H} \lambda_{H H}\left(1-\lambda_{H H}\right)}{t_{F H} \lambda_{H H}+1}\right)} \frac{\left(1-\lambda_{H H}\right)^{2}}{t_{F H} \lambda_{H H}+1}\left(\frac{\theta \sigma}{\sigma-1}-1\right) \frac{d t_{F H}}{1+t_{F H}} \\
& =\frac{\frac{1-\gamma}{\gamma}\left(1-\lambda_{F F}\right)\left(\frac{\lambda_{H H}\left(1+t_{F H}\right)}{1-\lambda_{H H}}+1\right)+\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F} \frac{\lambda_{H H}\left(1+t_{F H}\right)+1-\lambda_{H H}}{1-\lambda_{H H}}+1+\frac{1-\gamma}{\gamma} \frac{\frac{\theta-(\sigma-1)}{\sigma-1}}{\sigma-1}}{1+\left(\frac{\theta \sigma}{\sigma-1}-1\right)\left(\lambda_{F F}+\frac{1+t_{F H}}{\sigma_{F H} \lambda_{H H}+1} \lambda_{H H}\right)+\frac{1-\gamma}{\gamma}\left(2-\lambda_{H H}-\lambda_{F F}+\frac{\theta-(\sigma-1)}{\sigma-1} \frac{t_{F H} \lambda_{H H}\left(1-\lambda_{H H}\right)}{t_{F H} \lambda_{H H}+1}\right)} \frac{\left(1-\lambda_{H H}\right)^{2}}{t_{F H} \lambda_{H H}+1}\left(\frac{\theta \sigma}{\sigma-1}-1\right) \frac{d t_{F H}}{1+t_{F H}} \\
& =\frac{\frac{1-\gamma}{\gamma}\left(1-\lambda_{F F}\right)\left(\frac{\lambda_{H H}\left(1+t_{F H}\right)}{1-\lambda_{H H}}+1\right)+\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F} \frac{\lambda_{H H}+\lambda_{H H} t_{F H}+1-\lambda_{H H}}{1-\lambda_{H H}}+1+\frac{1-\gamma}{\gamma} \frac{\frac{\theta-(\sigma-1)}{\sigma-1}}{\sigma-1}}{1+\left(\frac{\theta \sigma}{\sigma-1}-1\right)\left(\lambda_{F F}+\frac{1+t_{F H}}{t_{F H} \lambda_{H H}+1} \lambda_{H H}\right)+\frac{1-\gamma}{\gamma}\left(2-\lambda_{H H}-\lambda_{F F}+\frac{\theta-(\sigma-1)}{\sigma-1} \frac{t_{F H} \lambda_{H H}\left(1-\lambda_{H H}\right)}{t_{F H} \lambda_{H H}+1}\right)} \frac{\left(1-\lambda_{H H}\right)^{2}}{t_{F H} \lambda_{H H}+1}\left(\frac{\theta \sigma}{\sigma-1}-1\right) \frac{d t_{F H}}{1+t_{F H}} \\
& =\frac{\frac{1-\gamma}{\gamma}\left(1-\lambda_{F F}\right)\left(\frac{\lambda_{H H}\left(1+t_{F H}\right)}{1-\lambda_{H H}}+1\right)+\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F} \frac{\lambda_{H H} t_{F H}+1}{1-\lambda_{H H}}+1+\frac{1-\gamma}{\gamma} \frac{\frac{\theta-(\sigma-1)}{\sigma-1}}{\sigma \sigma-1}}{1+\left(\frac{\theta \sigma}{\sigma-1}-1\right)\left(\lambda_{F F}+\frac{1+t_{F H}}{t_{F H} \lambda_{H H}+1} \lambda_{H H}\right)+\frac{1-\gamma}{\gamma}\left(\frac{1}{\theta}\left(2-\lambda_{H H}-\lambda_{F F}\right)+\frac{\theta-(\sigma-1)}{\sigma-1} \frac{t_{F H} \lambda_{H H}\left(1-\lambda_{H H}\right)}{t_{F H} \lambda_{H H}+1}\right)} \frac{\left(1-\lambda_{H H}\right)^{2}}{t_{F H} \lambda_{H H}+1}\left(\frac{\theta \sigma}{\sigma-1}-1\right) \frac{d t_{F H}}{1+t_{F H}} \\
& =\frac{\frac{1-\gamma}{\gamma}\left(1-\lambda_{F F}\right)\left(\frac{\lambda_{H H}\left(1+t_{F H}\right)}{1-\lambda_{H H}}+1\right)+\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F} \frac{\lambda_{H H} t_{F H}+1}{1-\lambda_{H H}}+1+\frac{1-\gamma}{\gamma} \frac{\theta-(\sigma-1)}{\theta-(\sigma-1)}}{1+\left(\frac{\theta \sigma}{\sigma-1}-1\right)\left(\lambda_{F F}+\frac{1+t_{F H}}{t_{F H} \lambda_{H H}+1} \lambda_{H H}\right)+\frac{1-\gamma}{\gamma}\left(\frac{1}{\theta}\left(2-\lambda_{H H}-\lambda_{F F}\right)+\frac{\theta-(\sigma-1)}{\sigma-1} \frac{t_{F H} \lambda_{H H}\left(1-\lambda_{H H}\right)}{t_{F H} \lambda_{H H}+1}\right)} \frac{\left(1-\lambda_{H H}\right)^{2}}{t_{F H} \lambda_{H H}+1}\left(\frac{\theta \sigma}{\sigma-1}-1\right) \frac{d t_{F H}}{1+t_{F H}} \\
& =\frac{\frac{1-\gamma}{\gamma}\left[\left(1-\lambda_{F F}\right)\left(\frac{\lambda_{H H}\left(1+t_{F H}\right)}{1-\lambda_{H H}}+1\right)+\frac{\theta-(\sigma-1)}{\theta \sigma-(\sigma-1)}\right]+\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F} \frac{\lambda_{H H} t_{F H}+1}{1-\lambda_{H H}}+1}{1+\left(\frac{\theta \sigma}{\sigma-1}-1\right)\left(\lambda_{F F}+\frac{1+t_{F H}}{t_{F H} \lambda_{H H}+1} \lambda_{H H}\right)+\frac{1-\gamma}{\gamma}\left(2-\lambda_{H H}-\lambda_{F F}+\frac{\theta-(\sigma-1)}{\sigma-1} \frac{t_{F H} \lambda_{H H}\left(1-\lambda_{H H}\right)}{t_{F H} \lambda_{H H}+1}\right)} \frac{\left(1-\lambda_{H H}\right)^{2}}{t_{F H} \lambda_{H H}+1}\left(\frac{\theta \sigma}{\sigma-1}-1\right) \frac{d t_{F H}}{1+t_{F H}} \\
& =\frac{\frac{1-\gamma}{\gamma} \frac{1-\lambda_{H H}}{t_{F H} \lambda_{H H}+1}\left[\left(1-\lambda_{F F}\right) \frac{\lambda_{H H}\left(1+t_{F H}+1-\lambda_{H H}\right.}{1-\lambda_{H H}}+\frac{\theta-(\sigma-1)}{\theta \sigma-(\sigma-1)}\right]+\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F}+\frac{1-\lambda_{H H}}{t_{F H} \lambda_{H H}+1}}{1+\left(\frac{\theta \sigma}{\sigma-1}-1\right)\left(\lambda_{F F}+\frac{1+t_{F H}}{t_{F H} \lambda_{H H}+1} \lambda_{H H}\right)+\frac{1-\gamma}{\gamma}\left(2-\lambda_{H H}-\lambda_{F F}+\frac{\theta-(\sigma-1)}{\sigma-1} \frac{t_{F H} \lambda_{H H}\left(1-\lambda_{H H}\right)}{t_{F H} \lambda_{H H}+1}\right)}\left(1-\lambda_{H H}\right)\left(\frac{\theta \sigma}{\sigma-1}-1\right) \frac{d t_{F H}}{1+t_{F H}} \\
& =\frac{\frac{1-\gamma}{\gamma} \frac{1-\lambda_{H H}}{t_{F H} \lambda_{H H}+1}\left[\left(1-\lambda_{F F}\right) \frac{\lambda_{H H}+\lambda_{H H} t_{F H}+1-\lambda_{H H}}{1-\lambda_{H H}}+\frac{\theta-(\sigma-1)}{\theta \sigma-(\sigma-1)}\right]+\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F}+\frac{1-\lambda_{H H}}{t_{F H} \lambda_{H H}+1}}{1+\left(\frac{\theta \sigma}{\sigma-1}-1\right)\left(\lambda_{F F}+\frac{1+t_{F H}}{t_{F H} \lambda_{H H}+1} \lambda_{H H}\right)+\frac{1-\gamma}{\gamma}\left(2-\lambda_{H H}-\lambda_{F F}+\frac{\theta-(\sigma-1)}{\sigma-1} \frac{t_{F H} \lambda_{H H}\left(1-\lambda_{H H}\right)}{t_{F H} \lambda_{H H}+1}\right)}\left(1-\lambda_{H H}\right)\left(\frac{\theta \sigma}{\sigma-1}-1\right) \frac{d t_{F H}}{1+t_{F H}} \\
& =\frac{\frac{1-\gamma}{\gamma} \frac{1-\lambda_{H H}}{t_{F H} \lambda_{H H}+1}\left[\left(1-\lambda_{F F}\right) \frac{\lambda_{H H} t_{F H}+1}{1-\lambda_{H H}}+\frac{\theta-(\sigma-1)}{\theta \sigma-(\sigma-1)}\right]+\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F}+\frac{1-\lambda_{H H}}{t_{F H} \lambda_{H H}+1}}{1+\left(\frac{\theta \sigma}{\sigma-1}-1\right)\left(\lambda_{F F}+\frac{1+t_{F H}}{t_{F H} \lambda_{H H}+1} \lambda_{H H}\right)+\frac{1-\gamma}{\gamma}\left(2-\lambda_{H H}-\lambda_{F F}+\frac{\theta-(\sigma-1)}{\sigma-1} \frac{t_{F H} \lambda_{H H}\left(1-\lambda_{H H}\right)}{t_{F H} \lambda_{H H}+1}\right)}\left(1-\lambda_{H H}\right)\left(\frac{\theta \sigma}{\sigma-1}-1\right) \frac{d t_{F H}}{1+t_{F H}} \\
& =\frac{\frac{1-\gamma}{\gamma}\left[1-\lambda_{F F}+\frac{\theta-(\sigma-1)}{\theta \sigma-(\sigma-1)} \frac{1-\lambda_{H H}}{t_{F H} \lambda_{H H}+1}\right]+\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F}+\frac{1-\lambda_{H H}}{t_{F H} \lambda_{H H}+1}}{1+\left(\frac{\theta \sigma}{\sigma-1}-1\right)\left(\lambda_{F F}+\frac{1+t_{F H}}{t_{F H} \lambda_{H H}+1} \lambda_{H H}\right)+\frac{1-\gamma}{\gamma}\left(2-\lambda_{H H}-\lambda_{F F}+\frac{\theta-(\sigma-1)}{\sigma-1} \frac{t_{F H} \lambda_{H H}\left(1-\lambda_{H H}\right)}{t_{F H} \lambda_{H H}+1}\right)}\left(1-\lambda_{H H}\right)\left(\frac{\theta \sigma}{\sigma-1}-1\right) \frac{d t_{F H}}{1+t_{F H}}
\end{aligned}
$$

First-order condition. The first-order condition reads

$$
\begin{aligned}
\frac{d W_{H}}{W_{H}}= & \frac{d \mu_{H}}{\mu_{H}}+\frac{\theta-(\sigma-1)}{\gamma \theta(\sigma-1)} \frac{d \tilde{\mu}_{H}}{\tilde{\mu}_{H}}-\frac{1}{\gamma \theta} \frac{d \lambda_{H H}}{\lambda_{H H}} \\
= & \frac{1}{1-(1-\gamma) \frac{\sigma-1}{\sigma}} \frac{\left(\frac{1}{\mu_{H}}-\lambda_{H H}+(1-\gamma) \frac{\sigma-1}{\sigma} \frac{\mu_{H}-1}{\mu_{H}} \lambda_{H H}\right) d t_{F H}-\left(1-(1-\gamma) \frac{\sigma-1}{\sigma} \frac{\mu_{H}-1}{\mu_{H}}\right) \lambda_{H H} t_{F H} \frac{d \lambda_{H H}}{\lambda_{H H}}}{\lambda_{H H} t_{F H}+1} \\
& +\frac{\theta-(\sigma-1)}{\gamma \theta(\sigma-1)}\left(\frac{1-\lambda_{H H}}{t_{F H} \lambda_{H H}+1} \frac{d t_{F H}}{1+t_{F H}}-\frac{t_{F H} \lambda_{H H}}{t_{F H} \lambda_{H H}+1} \frac{d \lambda_{H H}}{\lambda_{H H}}\right)-\frac{1}{\gamma \theta} \frac{d \lambda_{H H}}{\lambda_{H H}} \\
= & \frac{1}{1-(1-\gamma) \frac{\sigma-1}{\sigma}} \frac{\frac{1}{\mu_{H}}-\lambda_{H H}+(1-\gamma) \frac{\sigma-1}{\sigma} \frac{\mu_{H}-1}{\mu_{H}} \lambda_{H H}}{\lambda_{H H} t_{F H}+1} d t_{F H}+\frac{\theta-(\sigma-1)}{\gamma \theta(\sigma-1)} \frac{1-\lambda_{H H}}{t_{F H} \lambda_{H H}+1} \frac{d t_{F H}}{1+t_{F H}} \\
& -\frac{1-(1-\gamma) \frac{\sigma-1}{\sigma} \frac{\mu_{H}-1}{\mu_{H}}}{1-(1-\gamma) \frac{\sigma-1}{\sigma}} \frac{\lambda_{H H} t_{F H}}{\lambda_{H H} t_{F H}+1} \frac{d \lambda_{H H}}{\lambda_{H H}}-\frac{\theta-(\sigma-1)}{\gamma \theta(\sigma-1)} \frac{t_{F H} \lambda_{H H}}{t_{F H} \lambda_{H H}+1} \frac{d \lambda_{H H}}{\lambda_{H H}}-\frac{1}{\gamma \theta} \frac{d \lambda_{H H}}{\lambda_{H H}} \\
= & \left(\frac{\frac{1}{\mu_{H}}-\lambda_{H H}+(1-\gamma) \frac{\sigma-1}{\sigma} \frac{\mu_{H}-1}{\mu_{H}} \lambda_{H H}}{1-(1-\gamma) \frac{\sigma-}{\sigma}}+\frac{\theta-(\sigma-1)}{\gamma \theta(\sigma-1)} \frac{1-\lambda_{H H}}{1+t_{F H}}\right) \frac{d t_{F H}}{\lambda_{H H} t_{F H}+1} \\
& -\left(\frac{1-(1-\gamma) \frac{\sigma-1}{\sigma} \frac{\mu_{H}-1}{\mu_{H}}}{1-(1-\gamma) \frac{\sigma-1}{\sigma}} \frac{\lambda_{H H} t_{F H}}{\lambda_{H H} t_{F H}+1}+\frac{\theta-(\sigma-1)}{\gamma \theta(\sigma-1)} \frac{t_{F H} \lambda_{H H}}{t_{F H} \lambda_{H H}+1}+\frac{1}{\gamma \theta}\right) \frac{d \lambda_{H H}}{\lambda_{H H}} \\
= & \left(\frac{\frac{1}{\mu_{H}}-\lambda_{H H}\left(1-(1-\gamma) \frac{\sigma-1}{\sigma} \frac{\mu_{H}-1}{\mu_{H}}\right)}{1-(1-\gamma) \frac{\sigma-1}{\sigma}}+\frac{\theta-(\sigma-1)}{\gamma \theta(\sigma-1)} \frac{1-\lambda_{H H}}{1+t_{F H}}\right) \frac{d t_{F H}}{\lambda_{H H} t_{F H}+1} \\
& -\left(\left(\frac{1-(1-\gamma) \frac{\sigma-1}{\sigma} \frac{\mu_{H}-1}{\mu_{H}}}{1-(1-\gamma) \frac{\sigma-1}{\sigma}}+\frac{\theta-(\sigma-1)}{\gamma \theta(\sigma-1)}\right) \frac{\lambda_{H H} t_{F H}}{\lambda_{H H} t_{F H}+1}+\frac{1}{\gamma \theta}\right) \frac{d \lambda_{H H}}{\lambda_{H H}} .
\end{aligned}
$$

The role of $\gamma$ in the absence of selection. Evaluated at $t_{F H}=0$ (and therefor $\mu_{H}=1$ ), we have

$$
\begin{aligned}
\left.\frac{d W_{H} / W_{H}}{d t_{F H}}\right|_{t_{F H=0}}= & \left(\frac{\frac{1}{\mu_{H}}-\lambda_{H H}\left(1-(1-\gamma) \frac{\sigma-1}{\sigma} \frac{\mu_{H}-1}{\mu_{H}}\right)}{1-(1-\gamma) \frac{\sigma-1}{\sigma}}+\frac{\theta-(\sigma-1)}{\gamma \theta(\sigma-1)} \frac{1-\lambda_{H H}}{1+t_{F H}}\right) \frac{1}{\lambda_{H H} t_{F H}+1} \\
& -\left(\left(\frac{1-(1-\gamma) \frac{\sigma-1}{\sigma} \frac{\mu_{H}-1}{\mu_{H}}}{1-(1-\gamma) \frac{\sigma-1}{\sigma}}+\frac{\theta-(\sigma-1)}{\gamma \theta(\sigma-1)}\right) \frac{\lambda_{H H} t_{F H}}{\lambda_{H H} t_{F H}+1}+\frac{1}{\gamma \theta}\right) \frac{d \lambda_{H H} / \lambda_{H H}}{d t_{F H}} \\
= & \left(\frac{1-\lambda_{H H}}{1-(1-\gamma) \frac{\sigma-1}{\sigma}}+\frac{\theta-(\sigma-1)}{\gamma \theta(\sigma-1)}\left(1-\lambda_{H H}\right)\right)-\frac{1}{\gamma \theta} \frac{d \lambda_{H H} / \lambda_{H H}}{d t_{F H}} \\
= & \left(\frac{1}{1-(1-\gamma) \frac{\sigma-1}{\sigma}}+\frac{\theta-(\sigma-1)}{\gamma \theta(\sigma-1)}\right)\left(1-\lambda_{H H}\right)-\frac{1}{\gamma \theta} \frac{d \lambda_{H H} / \lambda_{H H}}{d t_{F H}}
\end{aligned}
$$

and
$\frac{d \lambda_{H H} / \lambda_{H H}}{d t_{F H}}=\frac{\frac{1-\gamma}{\gamma}\left[1-\lambda_{F F}+\frac{\theta-(\sigma-1)}{\theta \sigma-(\sigma-1)}\left(1-\lambda_{H H}\right)\right]+\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F}+1-\lambda_{H H}}{1+\left(\frac{\theta \sigma}{\sigma-1}-1\right)\left(\lambda_{F F}+\lambda_{H H}\right)+\frac{1-\gamma}{\gamma}\left(2-\lambda_{H H}-\lambda_{F F}\right)}\left(1-\lambda_{H H}\right)\left(\frac{\theta \sigma}{\sigma-1}-1\right)$

## Hence,

$$
\begin{aligned}
\left.\frac{1}{1-\lambda_{H H}} \frac{d W_{H} / W_{H}}{d t_{F H}}\right|_{t_{F H=0}}= & \frac{1}{1-(1-\gamma) \frac{\sigma-1}{\sigma}}+\frac{\theta-(\sigma-1)}{\gamma \theta(\sigma-1)}- \\
& \frac{\frac{\theta \sigma}{\sigma-1}-1}{\gamma \theta} \frac{\frac{1-\gamma}{\gamma}\left[1-\lambda_{F F}+\frac{\theta-(\sigma-1)}{\theta \sigma-(\sigma-1)}\left(1-\lambda_{H H}\right)\right]+\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda_{F F}+1-\lambda_{H H}}{1+\left(\frac{\theta \sigma}{\sigma-1}-1\right)\left(\lambda_{F F}+\lambda_{H H}\right)+\frac{1-\gamma}{\gamma}\left(2-\lambda_{H H}-\lambda_{F F}\right)}
\end{aligned}
$$

Assuming symmetry in the initial situation, we have

$$
\begin{aligned}
\left.\frac{1}{1-\lambda} \frac{d W_{H} / W_{H}}{d t_{F H}}\right|_{t_{F H=0}}= & \frac{1}{1-(1-\gamma) \frac{\sigma-1}{\sigma}}+\frac{\theta-(\sigma-1)}{\gamma \theta(\sigma-1)}- \\
& \frac{\frac{\theta \sigma}{\sigma-1}-1}{\gamma \theta} \frac{\frac{1-\gamma}{\gamma}(1-\lambda)\left[1+\frac{\theta-(\sigma-1)}{\theta \sigma-(\sigma-1)}\right]+\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda+1-\lambda}{1+2 \lambda\left(\frac{\theta \sigma}{\sigma-1}-1\right)+\frac{1-\gamma}{\gamma} 2(1-\lambda)}
\end{aligned}
$$

Additionally assuming absence of trade costs, we have $\lambda=0.5$ :

$$
\begin{aligned}
\left.\frac{1}{1-\lambda} \frac{d W_{H} / W_{H}}{d t_{F H}}\right|_{t_{F H=0}}= & \frac{1}{1-(1-\gamma) \frac{\sigma-1}{\sigma}}+\frac{\theta-(\sigma-1)}{\gamma \theta(\sigma-1)} \\
& -\frac{\frac{\theta \sigma}{\sigma-1}-1}{\gamma \theta} \frac{\frac{1-\gamma}{\gamma} \frac{1}{2}\left[1+\frac{\theta-(\sigma-1)}{\theta \sigma-(\sigma-1)}\right]+\left(\frac{\theta \sigma}{\sigma-1}-1\right) \frac{1}{2}+\frac{1}{2}}{1+2 \frac{1}{2}\left(\frac{\theta \sigma}{\sigma-1}-1\right)+\frac{1-\gamma}{\gamma} 2 \frac{1}{2}} \\
= & \frac{1}{1-(1-\gamma) \frac{\sigma-1}{\sigma}}+\frac{\theta-(\sigma-1)}{\gamma \theta(\sigma-1)} \\
& -\frac{1}{2} \frac{\frac{\theta \sigma}{\sigma-1}-1}{\gamma \theta} \frac{\frac{1-\gamma}{\gamma}\left[1+\frac{\theta-(\sigma-1)}{\theta \sigma-(\sigma-1)}\right]+\frac{\theta \sigma}{\sigma-1}-1+1}{1+\frac{\theta \sigma}{\sigma-1}-1+\frac{1-\gamma}{\gamma}} \\
= & \frac{1}{1-(1-\gamma) \frac{\sigma-1}{\sigma}}+\frac{\theta-(\sigma-1)}{\gamma \theta(\sigma-1)} \\
& -\frac{1}{2} \frac{\theta \sigma}{\sigma-1}-1 \frac{1-\gamma}{\gamma \theta}\left[1+\frac{\theta-(\sigma-1)}{\gamma \sigma-(\sigma-1)}\right]+\frac{\theta \sigma}{\sigma-1}
\end{aligned} \frac{\frac{\theta \sigma}{\sigma-1}+\frac{1-\gamma}{\gamma}}{}=
$$

Further assume $\theta \rightarrow \sigma-1$

$$
\begin{aligned}
\left.\frac{1}{1-\lambda} \frac{d W_{H} / W_{H}}{d t_{F H}}\right|_{t_{F H=0}} & =\frac{1}{1-(1-\gamma) \frac{\sigma-1}{\sigma}}-\frac{1}{2} \frac{\frac{(\sigma-1) \sigma}{\sigma-1}-1}{\gamma(\sigma-1)} \frac{\frac{1-\gamma}{\gamma}+\frac{\theta \sigma}{\sigma-1}}{\frac{\theta \sigma}{\sigma-1}+\frac{1-\gamma}{\gamma}} \\
& =\frac{1}{1-(1-\gamma) \frac{\sigma-1}{\sigma}}-\frac{1}{2} \frac{\sigma-1}{\gamma(\sigma-1)} \\
& =\frac{1}{1-(1-\gamma) \frac{\sigma-1}{\sigma}}-\frac{1}{2 \gamma}=\frac{2 \gamma-1+(1-\gamma) \frac{\sigma-1}{\sigma}}{2 \gamma\left(1-(1-\gamma) \frac{\sigma-1}{\sigma}\right)} \\
& =\frac{2 \gamma-1+\frac{\sigma-1}{\sigma}-\gamma \frac{\sigma-1}{\sigma}}{2 \gamma\left(1-(1-\gamma) \frac{\sigma-1}{\sigma}\right)}=\frac{\gamma\left(2-\frac{\sigma-1}{\sigma}\right)-\frac{1}{\sigma}}{2 \gamma\left(1-(1-\gamma) \frac{\sigma-1}{\sigma}\right)} \\
& =\frac{\gamma\left(\frac{2 \sigma-\sigma+1}{\sigma}\right)-\frac{1}{\sigma}}{2 \gamma\left(1-(1-\gamma) \frac{\sigma-1}{\sigma}\right)}=\frac{\gamma\left(\frac{\sigma+1}{\sigma}\right)-\frac{1}{\sigma}}{2 \gamma\left(1-(1-\gamma) \frac{\sigma-1}{\sigma}\right)} \\
& =\frac{\gamma(\sigma+1)-1}{2 \gamma \sigma\left(1-(1-\gamma) \frac{\sigma-1}{\sigma}\right)}
\end{aligned}
$$

Hence,

$$
\left.\frac{d W_{H} / W_{H}}{d t_{F H}}\right|_{t_{F H=0}} \gtrless 0 \Leftrightarrow \gamma(\sigma+1) \gtrless 1 \Leftrightarrow \gamma \gtrless \frac{1}{\sigma+1} .
$$

The role of $\eta$. Recall that

$$
\begin{aligned}
\left.\frac{1}{1-\lambda} \frac{d W_{H} / W_{H}}{d t_{F H}}\right|_{t_{F H=0}}= & \frac{1}{1-(1-\gamma) \frac{\sigma-1}{\sigma}}+\frac{\theta-(\sigma-1)}{\gamma \theta(\sigma-1)} \\
& -\frac{\frac{\theta \sigma}{\sigma-1}-1}{\gamma \theta} \frac{\frac{1-\gamma}{\gamma}(1-\lambda)\left[1+\frac{\theta-(\sigma-1)}{\theta \sigma-(\sigma-1)}\right]+\left(\frac{\theta \sigma}{\sigma-1}-1\right) \lambda+1-\lambda}{1+2 \lambda\left(\frac{\theta \sigma}{\sigma-1}-1\right)+\frac{1-\gamma}{\gamma} 2(1-\lambda)}
\end{aligned}
$$

Consider the limiting case $\theta \rightarrow \sigma-1$

$$
\left.\frac{1}{1-\lambda} \frac{d W_{H} / W_{H}}{d t_{F H}}\right|_{t_{F H=0}}=\frac{1}{1-(1-\gamma) \frac{\sigma-1}{\sigma}}-\frac{1}{\gamma} \frac{\frac{1-\gamma}{\gamma}(1-\lambda)+(\sigma-1) \lambda+1-\lambda}{1+2 \lambda(\sigma-1)+\frac{1-\gamma}{\gamma} 2(1-\lambda)}
$$

Moreover, consider $\gamma=1 /(\sigma+1)$. Then,

$$
\begin{aligned}
\left.\frac{1}{1-\lambda} \frac{d W_{H} / W_{H}}{d t_{F H}}\right|_{t_{F H=0}} & =\frac{1}{1-\left(1-\frac{1}{\sigma+1}\right) \frac{\sigma-1}{\sigma}}-(\sigma+1) \frac{\frac{1-\gamma}{\gamma}(1-\lambda)+(\sigma-1) \lambda+1-\lambda}{1+2 \lambda(\sigma-1)+\frac{1-\gamma}{\gamma} 2(1-\lambda)} \\
& =\frac{\sigma+1}{2}\left(1-2 \frac{\frac{1-\gamma}{\gamma}(1-\lambda)+(\sigma-1) \lambda+1-\lambda}{1+2 \lambda(\sigma-1)+\frac{1-\gamma}{\gamma} 2(1-\lambda)}\right) \\
& =\frac{\sigma+1}{2} \frac{1+2 \lambda(\sigma-1)+\frac{1-\gamma}{\gamma} 2(1-\lambda)-\frac{1-\gamma}{\gamma} 2(1-\lambda)-2(\sigma-1) \lambda-2(1-\lambda)}{1+2 \lambda(\sigma-1)+\frac{1-\gamma}{\gamma} 2(1-\lambda)} \\
& =\frac{\sigma+1}{2} \frac{1-2(1-\lambda)}{1+2 \lambda(\sigma-1)+\frac{1-\gamma}{\gamma} 2(1-\lambda)} \\
& =\frac{(\sigma+1)(\lambda-0.5)}{1+2 \lambda(\sigma-1)+\frac{1-\gamma}{\gamma} 2(1-\lambda)} .
\end{aligned}
$$

This expression implies that with $\gamma=1 /(\sigma+1)$, which implies that laissez-faire is optimal with $\eta=1$, with $\eta<1 \Leftrightarrow \lambda>0.5$, a tariff would be optimal.


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[^1]:    ${ }^{1}$ Goldberg and Pavenik (2016) discuss this point in a recent manuscript prepared for the new Handbook of Commercial Policy.

[^2]:    ${ }^{2}$ The reason is that with standard CES preferences, "consumer surplus" and "profit destruction" distortions exactly cancel out (Baldwin, 2005). Markup pricing is not a problem as all firms charge the same markup such that consumer spending decisions are not distorted.
    ${ }^{3}$ Demidova and Rodríguez-Clare (2009) prove this for a "small" economy. Felbermayr et al. (2013) generalize the result to two large countries. Jung (2012) shows for a small open economy, subsidies on operating (domestic) fixed costs financed by a lump-sum-tax on labor are also welfare improving.
    ${ }^{4}$ Caliendo et al. (2015) show that trade taxes generate entry effects in the presence of multiple sectors.

[^3]:    ${ }^{5}$ Note that this distortion is not present in the in Eaton and Kortum (2002) model where input producers produce under perfect competition.
    ${ }^{6}$ Caliendo et al. (2015) only consider import tariffs. Demidova and Rodríguez-Clare (2009) also consider domestic consumption tax-cum-subsidies.
    ${ }^{7}$ The model is flexible enough to allow for entry effects (which are different from selection effects). It turns out that while the mass of potential entrants depends on the degree of input-output linkages, it is not affected by the consumption subsidy. This finding suggests that the mass of potential entrants is socially optimal also in the version of the Melitz (2003) model that allows for input-output linkages. We have also experimented with a subsidy on the use of the composite input into production. It turns out that this policy is not welfare enhancing. In particular, it affects the mass of potential entrants. A labor tax is not suited to correct for the input distortion. To see this, consider the closed-economy case. A labor tax would directly affect the price of workers, but also affect the price of each good and therefore the price of the composite good, such that the relative price of workers is independent of the labor tax.

[^4]:    ${ }^{8}$ A policy addressing only domestic varieties is also not optimal because this affects relative spending of consumers on domestically produced goods; see below.

[^5]:    ${ }^{9}$ Demidova and Rodríguez-Clare (2009) consider a small open economy with heterogeneous firms. Haaland and Venables (2014) introduce an "outside sector" into the small open economy framework, which gives rise to another domestic distortion (markup distortion). Demidova (2015) explores the role of variable markups for optimal tariffs in a version of the Melitz and Ottaviano (2008) model that does away with the outside sector.

[^6]:    ${ }^{10}$ Caliendo et al. (2015) focus on the role of import tariffs, ruling out domestic consumption subsidies. They present a version of the model with two sectors of the kind presented here. A subsidy on the "consumption" of imported varieties is equivalent to an import subsidy.
    ${ }^{11}$ Blanchard et al. (2016) take a different stance by assuming that only input trade is subject to trade policy.

[^7]:    ${ }^{12}$ Recall that, to avoid confusion, we use the term intermediates when referring to final goods assembly, and the term intermediate inputs when referring to production.
    ${ }^{13}$ Imbruno (2014) considers more complex situations where sourcing from abroad also implies fixed costs of importing.

[^8]:    ${ }^{14}$ Note that in the limiting case $\theta \rightarrow \sigma-1$, the selection channel is inactive.

[^9]:    ${ }^{15}$ Indeed, implicitly defining a price index for country $i$ 's imports from country $j$ through $\tilde{P}_{i j}^{1-\sigma}:=N_{j} \chi_{j i} \xi_{j i}$, it can be shown that $\lambda_{i j}=\left(\tilde{P}_{i j} / \tilde{P}_{j}\right)^{1-\sigma}$.

[^10]:    ${ }^{16}$ As standard, we normalize domestic trade costs to unity, $t_{j j}=1$ for all $j$.

[^11]:    ${ }^{17}$ Notice that in the absence of commercial policies, the gross output multipliers $\tilde{\mu}_{j}$ simplify to the production value multiplier $\kappa$ as expenditure shares add up to unity.

[^12]:    ${ }^{18}$ Recall that commercial policy has no bearing on the mass of entrants.

[^13]:    ${ }^{19} \mathrm{As}$ argued above, the output multiplier bears same elasticity as fixed market access costs.

[^14]:    ${ }^{20}$ It will become clear below that the policy addressing domestic rather than imported goods has almost symmetric effects, the difference being that in the welfare calculus domestic expenditure shares have to be replaced by import expenditure shares, and vice versa.

[^15]:    ${ }^{21}$ Formally, this result follows from noting that $\mathrm{d} \mu / \mu=\left(t_{i}+\kappa^{-1}\right)^{-1} / \mathrm{d} t$.
    ${ }^{22}$ Dhingra and Morrow (2012) prove that market outcomes in the standard Melitz (2003) model with CES preferences are efficient, while in the presence of preferences that lead to variable markups, the efficiency result breaks. In a similar vein, Jung (2015) proves that the efficiency result does not carry over to the case of CESBenassy preferences. In this paper, we explore the sensitivity of the result to modifications on the production side of the economy.

[^16]:    ${ }^{23}$ We have shown above that commercial policy has no effect on the mass of entrants; see equation (38). Moreover, we have argued that with uniform treatment of domestically produced and imported varieties, commercial policy does not operate through the selection channel.
    ${ }^{24}$ The remaining workers operate as "fixed cost" workers. As the mass of active firms is fixed, labor requirement for fixed cost activities is also fixed.
    ${ }^{25}$ Formally, this follows from considering the limiting case $\theta \rightarrow \sigma-1$ in equation (40).

[^17]:    ${ }^{26}$ Notice that results for the opposite assumption can be obtained by a clever re-interpretation of expenditure shares.

[^18]:    ${ }^{27}$ It is also dependent on firm heterogeneity, but comparative statics results with respect to the labor cost share and the freeness of trade do not hinge on the selection effect being active.

[^19]:    ${ }^{28}$ Note that although Demidova and Rodríguez-Clare (2009) assume the country to be small, the government can affect "world market prices" by means of trade policy as in a monopolistically competitive setting, each firm is a monopolist in the particular variety it produces; see also Gros (1987) for the case of homogeneous firms.

[^20]:    ${ }^{29}$ Recall from above that in the limiting case $\theta \rightarrow \sigma-1$, laissez-faire is optimal at $\gamma=1 /(\sigma+1)$ when $\eta=1$.

